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"Liquidus Temperatures in Mixtures of the Feldspars of Soda, Potash, and Lime." By R. R. FRANCO and J. F. SCHAIRER

"The System Al_2O_3-H_2O." By GUY ERVIN, JR., and E. F. OSBORN

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THE JOURNAL OF GEOLOGY

May 1951

THE DISTRIBUTION OF Fe^{++} AND Mg^{++} IN COEXISTING OLIVINES AND PYROXENES[1]

HANS RAMBERG AND GEORGE DEVORE
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ABSTRACT

The Mg/(Mg + Fe) ratios of coexisting olivines and orthopyroxenes from different types of rocks have been determined by means of indices-of-refraction measurements. Some data from literature have also been added.

The distribution function of Mg and Fe in olivine-pyroxene pairs is discussed. It is shown that this function may be used as a geologic thermometer when some additional thermochemical data are obtained. The distribution function also indicates that orthopyroxene is a nonideal mixed crystal, the intermediate members of the series being more stable than one would expect. This nonideal character of the orthopyroxene series is indicated by deviation from linearity of the indices of refraction and the unit-cell dimensions when these quantities are plotted against the mol fraction of enstatite in the pyroxenes. The *a* and *b* parameters of the unit cell of different orthopyroxenes are measured. The aluminum content in orthopyroxenes is discussed briefly.

INTRODUCTION

The stability and equilibrium conditions of the minerals are most directly revealed by syntheses under controlled physicochemical conditions of the minerals in question. For metamorphic minerals this method meets severe practical difficulties because of the sluggishness of silicate reactions below the solidus temperature, and it is likely that we shall have to wait a long period of years before the complex equilibrium conditions of the rock-making minerals are explored thoroughly. In the meantime, field observations combined with theoretical thermodynamics may provide information about the equilibrium relations of metamorphic minerals. The present study of the distribution of Fe^{++} and Mg^{++} in coexisting olivines and orthopyroxenes represents this latter type of approach.

COMPOSITION OF COEXISTING PYROXENE AND OLIVINE

Table 1 and figure 2 give the composition of coexisting olivines and orthopyroxenes found in natural rocks, which, with two exceptions, are quartz-free. In figure 1 are compiled coexisting olivines and calcium-free clinopyroxenes from experimental work of Bowen and Schairer (1935, figs. 8, 9, 10, 11, 12, 13, 14). Because we are interested only in the distribution of Mg and Fe between coexisting rhombic pyroxene and olivine, independent of other phases which may be present, the complete Bowen and Schairer diagram is not reproduced.

There are some striking differences

[1] Manuscript received June 21, 1950.

TABLE 1*

COEXISTING OLIVINES AND ORTHOPYROXENES IN ROCKS

ROCK NO.	MOL FRACTION		OLIVINE	PYROXENE	ROCK TYPE	REFERENCE OR LOCALITY
	X_o	X_p	n_β ±0.002	n_γ ±0.002		
None	0.92	0.85	1.684	1.682	Sapphirine rock	Ramberg (1948, p. 17)
3835	.96	.83	1.660	1.687	Pyroxenite	Greenland
2030	.94	.78	1.668	1.693	Amphibolite sill	Greenland
2032	.94	.78	1.668	1.693	Amphibolite sill	Greenland
None	.92	.93	1.670	1.670	Olivine diopside pyrope rock	Eskola (1921, p. 22)
37066	.90	.93	1.672	1.671	Dunite	Greenland
12-56-C	.88	.79	1.676	1.690	Diabase dike	Wyoming
1682	.81	.80	1.693	Given	Gabbro picrite	Wager and Deer (1939, p. 159)
4087	.68	.70	1.739	1.705	Picrite	Wager and Deer (1939, p. 68)
6-21-N	.66	.72	1.725	1.703	Anorthosite	Wyoming
4077	.64	.63	1.749	1.715	Gabbro	Wager and Deer (1939, p. 68)
6-24-V	.64	.67	1.728	1.707	Anorthosite	Wyoming
TrII-50	.62	.63	1.736	1.715	Lamprophyre	Wyoming
6-98-D	.61	.67	1.737	1.707	Anorthosite	Wyoming
6-21-Q	.60	.64	1.738	1.713	Anorthosite	Wyoming
TrIIA-3A	.60	.65	1.738	1.710	Norite	Wyoming
5-37-D	.60	.63	1.738	1.715	Basic pegmatite	Wyoming
6-80-Ha	.56	.63	1.748	1.715	Anorthosite	Wyoming
IMI5-148	.55	.67	1.750	1.707	Ti-Fe ore	Wyoming
TrIID-7B	.54	.66	1.756	1.709	Lamprophyre	Wyoming
TrIIB-9C	.52	.65	1.758	1.710	Norite	Wyoming
T-137	.51	.59	1.760	1.719	Lamprophyre	Wyoming
TrII-4A	.42	.59	1.774	1.719	Anorthosite	Wyoming
12-37-Kb	.42	.59	1.774	1.719	Lamprophyre	Wyoming
Tr9-23A	.41	.63	1.778	1.715	Lamprophyre	Wyoming
12-37-F	.41	.64	1.778	1.713	Syenite	Wyoming
5-24-F	.41	.64	1.778	1.713	Fe-Ti ore	Wyoming
119	.39	.51	1.780	($2V = 52°–54°$)	Dark lava	Barth (1939, p. 10)
122	.39	.51	1.780	1.730	Dark lava	Barth (1939, p. 10)
12-37-KD	.37	.54	1.786	1.726	Syenite	Wyoming
6-25-37a	.36	.48	1.780†	1.735	Norite	Wyoming
5-24-Ia	.35	.51	1.792†	1.730	Fe-Ti ore	Wyoming
6-25-36B	.32	.59	1.797†	1.719	Lamprophyre	Wyoming
12-37-Ke	.24	.54	1.810†	1.726	Ti-Fe ore	Wyoming
5-24-Fa	.19	.49	1.826†	1.732	Syenite	Wyoming
12-37-Kd	.19	.43	1.826†	1.738	Syenite	Wyoming
5-24-Fd	.16	.47	1.795‡	1.735	Syenite	Wyoming
12-37-I	.14	.44	1.798‡	1.737	Syenite	Wyoming
TrI-62	.14	.57	1.798‡	1.728	Syenite	Wyoming
5-25-A	.14	.45	1.798‡	1.735	Syenite	Wyoming
5-24-G	.13	.45	1.800‡	1.736	Syenite	Wyoming
12-37-G	.13	.40	1.800‡	1.744	Syenite	Wyoming
7-42-H	.12	.57	1.802‡	1.728	Syenite	Wyoming
4-30-D	.10	.47	1.804‡	1.735	Syenite	Wyoming
6-44-Cl	.09	.54	1.806‡	1.726	Syenite	Wyoming
6-44-C	.07	.47	1.812‡	1.735	Syenite	Wyoming
TrII-55	0.07	0.39	1.812‡	1.745	Syenite	Wyoming

* Greenland specimens were collected by H. Ramberg; Wyoming specimens were collected by W. H. Newhouse, A. F. Hagner, and G. W. DeVore from the Laramie Range. From the Laramie Range complex, many duplications of the above listed determinations were observed. In the mineral pair $X_o = 0.65–0.63$ with $X_p = 0.65–0.67$ there are twelve additional examples. In the pair $X_o = 0.61–0.62$ with $X_p = 0.66–0.67$ there are seven additional examples. Most of the other determinations are duplicated at least once. Material from the literature is included only if it was definite that the two minerals were in the same hand specimen and if the analysis or index of refraction was listed. All composition values were determined from the index of refraction curves in Kennedy (1947, pp. 564 and 567).

† ±0.004.

‡ alpha ±0.004.

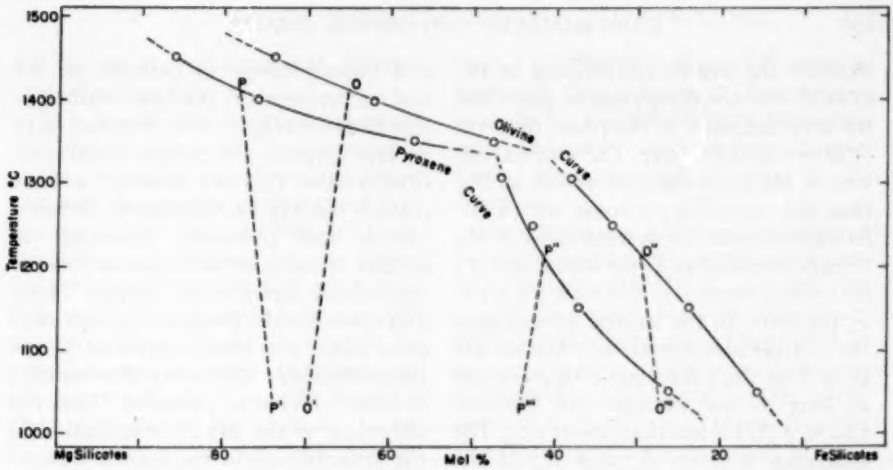

FIG. 1.—Compositions of artificial olivines coexisting with Ca-free clinopyroxenes (from the work of Bowen and Schairer, 1935). The break between the curves is caused by the existence of a melt, with the minerals on the left half of the diagram and with only solid phases on the right half. The dashed lines, p–p', p''–p''', o–o', and o''–o''', represent compositional changes with temperature if free silica is absent.

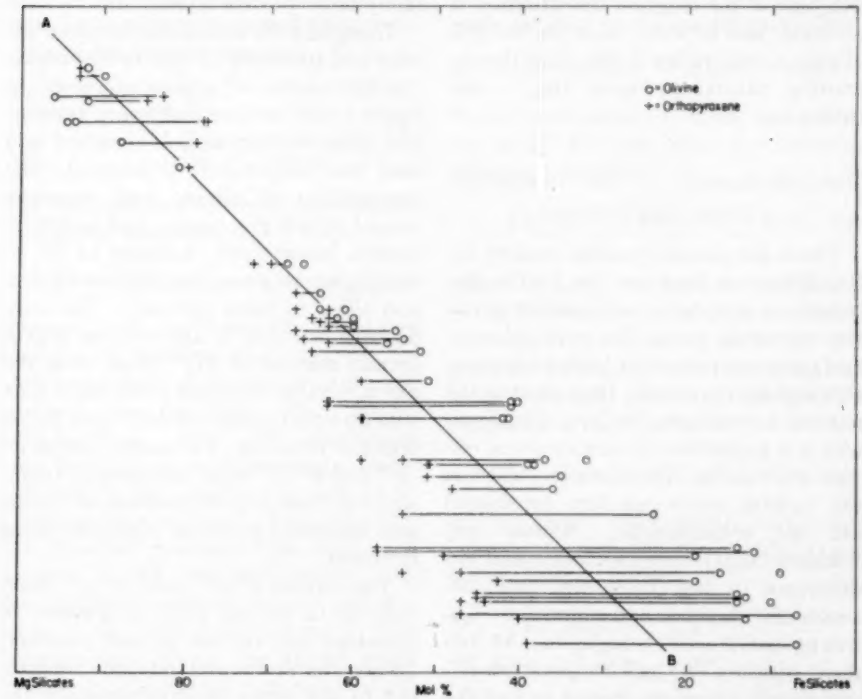

FIG. 2.—Compositions of natural olivines coexisting with orthopyroxenes of table 1, plotted so that the mean of each coexisting pair falls on the line AB. Line AB forms an angle of 45° with the base of the diagram.

between the Mg-Fe distribution in the natural olivine-orthopyroxene pairs and the artificial pairs in the phase diagram of Bowen and Schairer. The artificial olivine is always somewhat poorer in Mg than the coexisting pyroxene, with a difference between the mol fractions of Mg silicate (denoted as X_o for olivine and X_p for orthopyroxene) of not more than 10–15 per cent. In the natural assemblages the olivines also contain less Mg-silicate than does the coexisting orthopyroxene as long as the average mol fraction, $(X_o + X_p)/2$ is less than about 0.65. The difference between X_o and X_p of the natural coexisting minerals is, on the average, considerably greater than in the artificial system within the more Fe-rich portion of the diagram. Within the Mg-rich part of the diagram the situation is reversed, and in some cases the natural olivine is even richer in Mg than the coexisting natural pyroxene (fig. 2 and table 1).

THERMODYNAMICS OF THE PYROXENE-OLIVINE ASSEMBLAGES

There are several possible reasons for the difference between the Mg-Fe distributions in natural and in artificial olivine-pyroxene pairs. The artificial mineral pairs are formed at higher temperature and lower pressure than most of the natural assemblages. In the artificial system it is a question of coexistence of olivine and Ca-free clinopyroxene, whereas the natural pyroxenes here considered are all orthorhombic. Bowen and Schairer (1935) have concluded that the difference in Mg/(Fe + Mg) ratio between the rhombic and monoclinic Ca-free pyroxene series is negligible. At constant pressure (P) and temperature (T) one should, therefore, expect to find the same Mg-Fe distribution between olivine and clinopyroxene as between olivine and orthopyroxene. Natural olivine-pyroxene assemblages occur together with several minerals not present in the artificial system. This fact, however, will not disturb the Mg-Fe distribution between olivine and pyroxene, inasmuch as neither of the minerals tends to dissolve appreciable amounts of "foreign" ions; and therefore the presence of other minerals does not affect appreciably the thermodynamic properties (free energy) of either olivine or pyroxene. Thus the difference of the Mg-Fe distribution in the artificial and in the natural olivine-pyroxene pairs should be mainly due to temperature and/or pressure differences, although some of the natural coexisting olivine-orthopyroxene pairs may certainly not represent equilibrium.

To explain the interaction between olivine and pyroxene in quartz-free rocks, consider olivine o'' and pyroxene p'' in figure 1 and assume that every trace of free silica or tridymite is removed and that the temperature is lowered. The composition of olivine and pyroxene cannot follow the olivine and pyroxene curves, respectively, because to do so would require a reaction between olivine and silica to form pyroxene. The only possible reaction, if any, will be that a certain amount of Mg^{++} goes from the one mineral to the other at the same time that an equal amount of Fe^{++} goes in the opposite direction. The dashed curves o''–o''' and p''–p''' may, for example, represent the changing composition of olivine and coexisting pyroxene when free silica is absent.

The curves o''–o''' and p''–p''' hold only for a certain ratio of amount of pyroxene and olivine present together. In case the olivine and pyroxene are present in any other relative amounts, the change of composition with temperature

will follow curves somewhat different from o''–o''' and p''–p'''.

In a quartz-free assemblage beneath the olivine (or pyroxene) curve in figure 1, the composition of olivine (or pyroxene) can change at a given temperature from pure forsterite (or enstatite) to the composition determined by the olivine (or pyroxene) curve at this given temperature. If both mineral series are ideal mixtures, there is a simple relation between the compositions of pyroxenes and olivines which are in equilibrium at a given temperature and pressure. In our following discussion we shall consider the olivine and pyroxene series to be ideal mixed crystals, as Sahama and Torgeson (1949) concluded, and under this preliminary assumption determine the character of the distribution constant and its relation to temperature and pressure.

Consider first the reaction:

$$\left.\begin{array}{l} \underset{\text{forsterite}}{Mg_2SiO_4} + 2\underset{\text{ferrosilite}}{FeSiO_3} \\ \qquad \rightarrow \underset{\text{fayalite}}{Fe_2SiO_4} + 2\underset{\text{enstatite}}{MgSiO_3}\,. \end{array}\right\} \quad (1)$$

The standard free energy of this reaction at a given temperature, T'° K. is:

$$\Delta F' = F'_{fay} + 2F'_{en} - F'_{for} - 2F'_{ferr}\,, \quad (2)$$

where F'_{fay}, F_{en}, etc., are the molal free energies of fayalite, enstatite, etc., in their pure state at the temperature and pressure in question. The olivine, $(Mg,Fe)_2SiO_4$, and the pyroxene, $(Mg,Fe)SiO_3$, are mixed crystals; therefore, the actual reaction cannot be so simple as that expressed in equation (1). The fayalite molecules that form will dissolve in the forsterite present, and the enstatite molecules formed will dissolve in the original ferrosilite. By this homogeneous mixing the partial molal free energy of forsterite in the olivine will decrease, whereas the partial molal free energy of fayalite in olivine will increase. Similarly, the partial molal free energies of enstatite and ferrosilite in the pyroxene will increase and decrease, respectively, when reaction (1) goes to the right.

Conditions for chemical equilibrium between olivine and pyroxene at a constant temperature T'° K. are that the sum of the partial molal free energies of forsterite in olivine and of ferrosilite in pyroxene is equal to the sum of the partial molal free energies of fayalite in olivine and of enstatite in pyroxene (eq. [3]). The two minerals are known to be ionic crystals in which the exchangeable constituents Fe^{++} and Mg^{++} occur in the same state in both minerals, namely, as individual ions. Under these conditions one may consider the reactions connected with one mol of the exchangeable ions:

$$MgSi_{1/2}O_2 + FeSiO_3 \rightarrow FeSi_{1/2}O_2 + MgSiO_3\,.$$

Equilibrium between olivine and pyroxene at T'° K. exists when the following relation is satisfied:

$$\tfrac{1}{2}\overline{F}'_{fay} + \overline{F}'_{en} - \tfrac{1}{2}\overline{F}'_{for} - \overline{F}'_{ferr} = 0\,, \quad (3)$$

where $\overline{F}'_{fay}$, $\overline{F}'_{en}$, etc., are now the partial molal free energies of fayalite in olivine, enstatite in pyroxene, etc., at T'° K. The general relationship between the partial molal free energy, $\overline{F}$, and the composition of an ideal mixed crystal is

$$\overline{F}_i = F_i + RT \ln X_i\,,$$

where F_i is the free energy of component i in its pure state. Hence equation (3) can be expressed as follows:

$$\left.\begin{array}{l} \tfrac{1}{2}F'_{fay} + RT' \ln (1 - X'_o) + F'_{en} \\ \quad + RT' \ln X'_p - \tfrac{1}{2}F'_{for} - RT' \ln X'_o \\ \quad - F'_{ferr} - RT' \ln (1 - X'_p) = 0\,, \end{array}\right\} \quad (4)$$

where R is the gas constant, X_o is the mol fraction of $MgSi_{1/2}O_2$ in olivine, X'_p is the mol fraction of $MgSiO_3$ in pyroxene, $1-X'_o$ is the mol fraction of $FeSi_{1/2}O_2$ in olivine, and $1-X'_p$ is the mol fraction of $FeSiO_3$ in pyroxene.[2]

Rearrangement of equation (4) gives:

$$\left.\begin{aligned}\ln\left(\frac{X'_p}{1-X'_p}\cdot\frac{1-X'_o}{X'_o}\right)\\ =\frac{\frac{1}{2}F'_{for}+F'_{ferr}-\frac{1}{2}F'_{fay}-F'_{en}}{RT'}\end{aligned}\right\}\quad(5)$$

or

$$\frac{X'_p}{1-X'_p}\cdot\frac{1-X'_o}{X'_o}=e^{-\Delta F'/2RT'}=K',\quad(6)$$

where $\Delta F'$ is the standard free energy of reaction (1).

Because $\Delta F'$ is constant at a constant temperature (and pressure), it follows that the term

$$\frac{X'_p}{1-X'_p}\cdot\frac{1-X'_o}{X'_o}$$

is constant at constant P and T, provided that the olivine and the pyroxene are both ideal mixed crystals. Hence, by knowing the X_p and X_o for one pair of stably coexisting olivine-pyroxene assemblages at a given temperature, the distribution constant,

$$\frac{X_p}{1-X_p}\cdot\frac{1-X_o}{X_o}=K,$$

is known and will give corresponding compositions of any stable pair of coexisting olivine-pyroxene assemblages at that temperature, provided, of course, that the two mineral series are ideal mixtures.

Applying the considerations above at any temperature, T, different from T', we find that the following relation is valid:

$$\left.\begin{aligned}\frac{X_p}{1-X_p}\cdot\frac{1-X_o}{X_o}=\left(\frac{X'_p}{1-X'_p}\cdot\frac{1-X'_o}{X'_o}\right)^{T'/T}\\ \times\, e^{(\Delta F'-\Delta F)/2RT},\end{aligned}\right\}\quad(7)$$

where

$$\frac{X_p}{1-X_p}\cdot\frac{1-X_o}{X_o}=K$$

is the distribution constant at any temperature, T;

$$\frac{X'_p}{1-X'_p}\cdot\frac{1-X'_o}{X'_o}=K'$$

is the distribution constant at a reference temperature T'; and $\Delta F'$ and ΔF are the standard free energies at T' and T, respectively. Now, assuming that the entropies of enstatite, forsterite, ferrosilite, and fayalite are constant within the temperature range $T'-T$, the standard free energy difference can be expressed as follows:

$$\Delta F'-\Delta F=\Delta S\,(T'-T),\quad(8)$$

where the entropy difference is

$$\Delta S=S_{for}+2S_{ferr}-(S_{fay}+2S_{en}).$$

This can be introduced into equation (7):

$$\left.\begin{aligned}\frac{X_p}{1-X_p}\cdot\frac{1-X_o}{X_o}=\left(\frac{X'_p}{1-X'_p}\cdot\frac{1-X'_o}{X'_o}\right)^{T'/T}\\ \times\, e^{\Delta S(T'-T)/2RT}.\end{aligned}\right\}\quad(9)$$

If the pyroxenes and olivines are ideal mixed crystals and ΔS were known, these minerals would provide a usable geologic thermometer. By using X'_o and X_p from the Bowen and Schairer experiments, we can get experimental values of the constant

$$K=\frac{X'_p}{1-X'_p}\cdot\frac{1-X'_o}{X'_o}$$

[2] It is here assumed for simplicity that the partial molal free energy of SiO_4 in the lattice is constant at varying Mg/(Mg,Fe) ratio in the olivine and pyroxene.

at known temperatures. Any stable natural assemblage of olivine and pyroxene would give a new distribution constant,

$$K = \frac{X_p}{1-X_p} \cdot \frac{1-X_o}{X_o},$$

which will enable us to measure the corresponding temperature at which the natural assemblage was "quenched."

However, from the data gathered in table 1 and plotted in figure 2, it can be concluded that the rhombic pyroxenes and/or the olivines are not ideal mixed crystals because K is not constant at constant T. The rocks compiled in table 1 and in figure 2, in all likelihood, were formed at different temperatures because they are partly lavas, partly plutonic igneous rocks, and partly metasomatic and metamorphic rocks. There is little reason to believe, though, that there is a pronounced relation between the temperature which the mineral represents and the average

$$X_m = \frac{X_o + X_p}{2}$$

in the olivine-pyroxene pairs in such a way, for example, that a high X_m represents high temperature or vice versa. So far, we have reason to believe, because of similar facies (granulite) existing across the diagram (fig. 2), that the Mg-rich minerals from Greenland are formed at a temperature similar to the temperature represented by the iron-rich minerals from the Laramie Range, Wyoming (table 1). We therefore think it is true that the temperature interval inside which the rocks (in table 1) have formed (or recrystallized) is independent of the average X_m of the coexisting olivine-pyroxene pairs. On the other hand, we see that, statistically, the $X_p/(1-X_p) \cdot (1-X_o)/X_o$ values of our rocks change regularly with the average X_m of the assemblages, the iron-rich assemblages having much greater K-values than the Mg-rich assemblages (table 2). The reason must be that the minerals concerned are not ideal mixed crystals, for which equations (7) and (9) should be valid. For this reason equations (7) and (9) have to be restricted to a certain narrow $(X_o + X_p)/2$ range when used as a temperature indicator. That is, only olivine-pyroxene pairs with a rather similar mean X should be used when the K's are compared in an åttempt to determine the temperature of "quenching."

TABLE 2

DISTRIBUTION CONSTANTS FROM NATURAL ROCKS

Specimen No.	X_o	X_p	X_m	K
3835.....	0.96	0.83	0.89	0.205
12-56-C....	.88	.79	.83	0.402
4087.....	.68	.70	.69	1.099
5-37-D.....	.60	.63	.61	1.135
12-37-Kb...	.42	.59	.50	1.987
122.....	.39	.51	.45	1.626
6.25-36b...	.32	.59	.45	3.059
12-37-Ke...	.24	.54	.39	3.716
12-37-G....	.13	.40	.26	4.460
5-24-Fd....	.16	.47	.31	4.655
6-44-Cl....	0.09	0.54	0.31	11.870

Before discussing the matter further, we shall consider the effect of pressure upon the distribution constant, K. At a constant temperature, T', and a given pressure, P', we have (eq. [5])

$$\ln K' = -\frac{\Delta F'}{2RT'}. \qquad (10)$$

Keeping T' constant but changing pressure to P will change $\Delta F'$ to ΔF and hence also K' to K_p:

$$\ln K_p = -\frac{\Delta F}{2RT'}. \qquad (11)$$

Equation (11) minus equation (10) gives

$$K_p = K' \, e^{(\Delta F' - \Delta F)/2RT'},$$

where $\Delta F' - \Delta F$ is the standard free energy difference caused by the pressure difference $P' - P$. Therefore,

$$\Delta F' - \Delta F = \Delta V\,(P' - P)\,,$$

where ΔV is the mol volume difference:

$$V_{fay} + 2\,V_{en} - (V_{for} + 2\,V_{ferr})\,.$$

The effect of pressure upon the distribution constant, therefore, is

$$K = K'\,e^{\Delta V(P'-P)/2RT'}.$$

We know approximate values of ΔV from the following values of densities, d, and mol volumes V:

Forsterite (Poldervaart, 1947):
$d = 3.19$ g/cc; $V = 44.10$ cc.
Fayalite (Poldervaart, 1947):
$d = 4.14$ g/cc; $V = 49.23$ cc.
Enstatite (Poldervaart, 1947):
$d = 3.18$ g/cc; $V = 31.57$ cc.
Ferrosilite (Birch, Schairer, and Spicer, 1942):
$d = 3.97$ g/cc; $V = 33.23$ cc.
$\Delta V = 1.81$ cc.

Although the density data are not entirely reliable and we do not know the compressibilities of the minerals, we shall see whether high pressure can have an important effect on the K-values. Assume that $P = 10{,}000$ atm. (more than 30-km. depth) and $P' = 0$ atm. The value of T' is, for example, 800° K. Therefore,

$$K = K'\,e^{-1.81\cdot 10{,}000/2\cdot 82.1\cdot 800},$$

where $R = 82.1$ atm. cc/degree. Therefore,

$$K = 0.87\,K'\,,$$

which means that the pressure sensitivity of the equilibrium constant is rather negligible, assuming that nothing serious happens to the mol volume under the increasing pressure. It is therefore likely that the main reason for the discrepancy between the K-values in the natural olivine-pyroxene assemblages and those from the Bowen and Schairer experiments is due primarily to temperature differences.

Although it appears that the complete olivine or pyroxene mixed crystal series cannot be ideal mixtures, one may reasonably assume that the pyroxenes behave like an ideal mixture within the rather narrow range $X_p = 0.2$–0.5, and olivine within $X_o = 0.05$–0.30. Within these compositions, equation (9) should apply fairly well and is thus a useful geologic thermometer. However, the important quantity, ΔS, is not known with the accuracy necessary to calculate the temperature of the natural assemblages. For the sake of order of magnitude, an attempt to estimate ΔS is made from the entropy data compiled in table 3. The resulting ΔS is 1.0 cal/mol/degree at 1,200° K. and Δ is 0.5 cal/mol/degree at 298° K. These values are certainly subject to rather great probable errors and do not indicate much more than that ΔS is close to zero or perhaps has a small positive value of the order of magnitude 1–3 cal/mol/degree.

There exists another possibility of calculating ΔS. From figure 1 the corresponding X_p and X_o are experimentally known over a temperature range. Provided that the data compiled from Bowen and Schairer in figure 1 are sufficiently accurate, the different K's which can be computed from corresponding X_p and X_o at different temperatures will give ΔS within this range of temperature. Table 4 gives corresponding X_p's, X_o's, K's, and the computed ΔS at the different temperatures. It is interesting to note that the ΔS thus calculated has a positive value of a magnitude rather comparable with that of the ΔS calculated from the experimental data in table 3.

In figure 3, K and ln K are plotted versus temperature. One could now plot

TABLE 3

DATA AVAILABLE FOR AN APPROXIMATE ESTIMATE OF ΔS OF EQUATION (9)*

MINERAL	ENTROPIES IN CAL/DEGREE/MOL		
	At 298°.1 K.	Between 298°.1 K. and 1,200° K.	Total at 1,200° K.
FeO	14.1	18.50±0.36	32.60±
MgO	6.55±0.15	14.98±0.30	21.53±
SiO_2 (q)	10.06±0.10	20.95±0.10	30.95±
SiO_2 (t)	10.36±0.15	20.72±0.41	31.08±
$MgSiO_3$	16.2 ±0.20	35.27±0.35	51.47±
Mg_2SiO_4	22.7 ±0.2		
Mg_2SiO_4	22.7 ±0.2	50.25 est†	72.95 est(q)‡; 72.95 est(t)§
Mg_2SiO_4	22.7 ±0.2	49.59 est‖	72.29 est(q); 72.52 est(t)
Mg_2SiO_4	22.7 ±0.2	50.91 est#	73.61 est(q); 73.28 est(t)
Mg_2SiO_4		Average est 50.25	72.95±(q); 72.95±(t)
Fe_2SiO_4	35.4 ±1.4	57.70±1.5	93.1±
$FeSiO_3$	21.3 est**		60.5±est(q); 60.5 est(t)
$FeSiO_3$	22.7 est††		62.03±est(q); 62.09 est(t)
$FeSiO_3$	24.1 est‡‡		63.6±est(q); 63.68 est(t)
$FeSiO_3$ (av. est)	22.7		62.04±(q); 62.09±(t)

* From Kelley (1941, 1943).

† $S_{tot_{298\text{-}1,200}} = S_{MgO_{298\text{-}1,200}} + S_{est_{298\text{-}1,200}}$.

‡ "est(q)" means that the entropy of quartz is used.

§ "est(t)" means that the entropy of tridymite is used.

‖ $S_{tot_{298.1\text{-}200}} = 2S_{est_{298\text{-}1,200}} - S_{SiO_{2298\text{-}1,200}}$.

$S_{tot_{298\text{-}1,200}} = 2S_{MgO_{298\text{-}1,200}} + S_{SiO_{2298\text{-}1,200}}$.

** $S_{tot} = S_{tot} - S_{FeO}$.

†† $S_{tot} = \frac{1}{2}(S_{tot} + S_{SiO_2})$.

‡‡ $S_{tot} = S_{SiO_2} + S_{FeO}$.

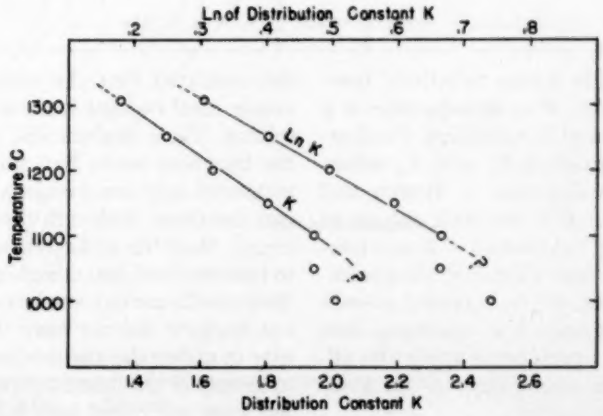

FIG. 3.—Diagram of temperature versus the distribution constants K and the natural logarithms of K from the composition interval of $X_o = 0.05$–0.30 and $X_p = 0.20$–0.50 of the artificial system of Bowen and Schairer (1935). Data are listed in table 4.

the K's from the natural olivine-pyroxene pairs (table 2), with $(X_p + X_o)/2$ values comparable with those of the synthetic minerals in table 4, on the extension of the K versus temperature curve in figure 3. Nos. 12-37-Ke, 12-37-G, 5-24-Fd, and 6-44-Cl in table 2 have the proper $(X_p + X_o)/2$ values. It may be seen that their K-values—3.716, 4.46, 4.655, and 11.87, respectively—will correspond to very low temperatures indeed, if the K or ln K versus temperature functions are linear.

X_p (and X_o) in the natural rocks. In particular, when X_p increases, K decreases strikingly, as is shown in table 2.

We have seen that there is every reason to believe that the temperature (and pressure) represented by the large number of rocks here compiled does not change in any regular manner with the mean X. Therefore, the regular change of K with mean X must be due to deviation from ideality of one or both of the two mixed crystals.

Sahama and Torgeson (1949) have

TABLE 4

DISTRIBUTION CONSTANTS AND ΔS FROM THE BOWEN AND SCHAIRER DIAGRAM

T° C.	T° K.	X_p	X_o	K	ln K	$\Delta S/2$ (Cal/Degree/Mol)
1,305	1,577	0.464	0.381	1.36	0.3075	
						+2.43
1,250	1,523	.426	.331	1.50	.4055	
						+2.22
1,200	1,473	.395	.285	1.63	.4947	
						+2.30
1,150	1,423	.361	.237	1.82	.5933	
						+1.25
1,100	1,373	.320	.195	1.95	.6678	
						+0.66
1,050	1,323	.260	.153	1.95	.6678	
						+1.20
1,000	1,273	0.205	0.110	2.10	0.7419	

There is little reason to believe, however, that either K or its logarithm is a linear function of te.nperature. Furthermore, the coexisting X_o and X_p values found in the diagrams of Bowen and Schairer may not be accurate enough to allow a correct calculation of K as a temperature function. Therefore, the presentations here should be regarded as only qualitative, inasmuch as too many data are lacking at present to apply the olivine-pyroxene assemblages as an absolute geothermometer.

DEVIATION FROM IDEALITY OF THE ENSTATITE-FERROSILITE MIXED CRYSTAL

Let us now return to the interesting feature that K is changing regularly with

demonstrated that the olivine series is nearly ideal because it shows no heat of mixing. These authors also investigated the pyroxene series but, unfortunately, measured only compositions close to the end members. Although they concluded (1949) that the orthopyroxenes appear to represent an ideal mixed-crystal series, their results are not conclusive. The present workers did not have the opportunity to undertake thermochemical measurements of the intermediate pyroxenes, but some additional facts indicating that the orthopyroxenes are not ideal mixed crystals will be emphasized.

The Fe-Mg distributions indicate that, as is usually suspected and was also demonstrated by Sahama and Torgeson, the iron-rich members of the pyroxenes

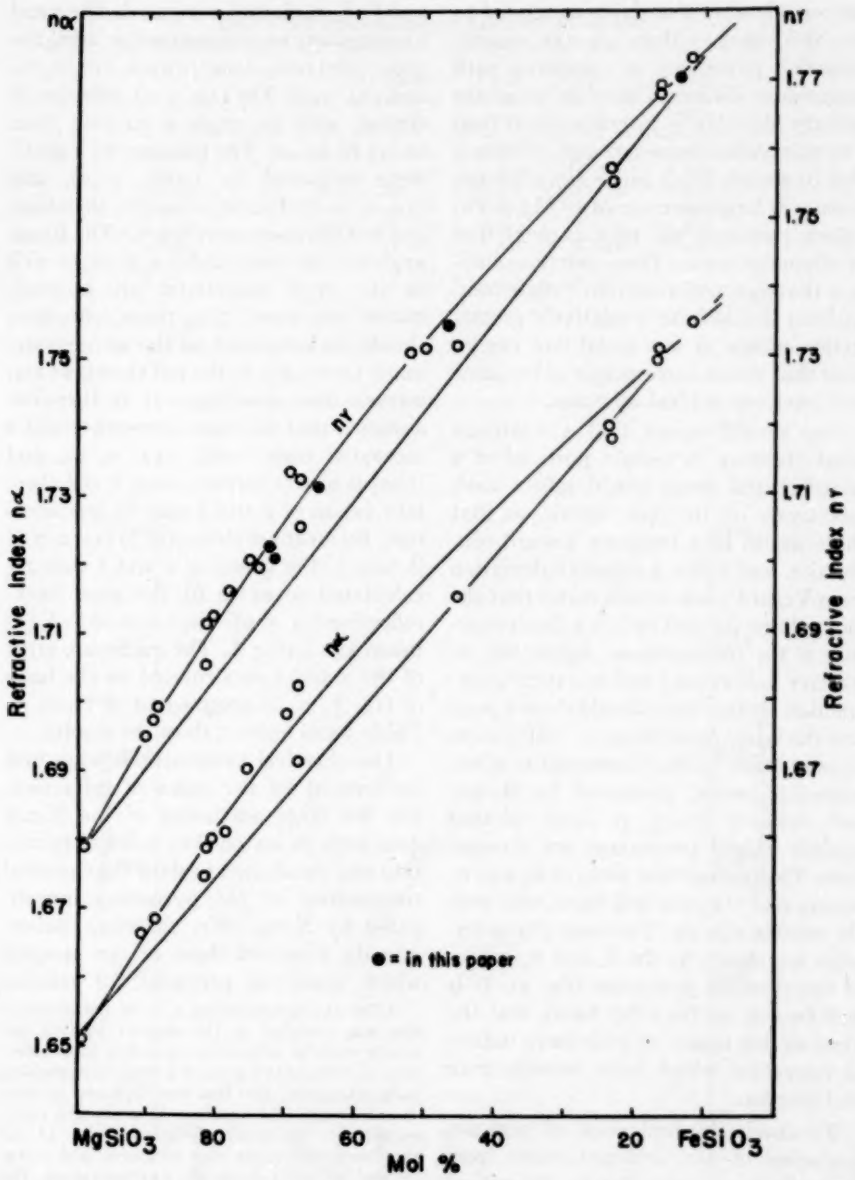

FIG. 4.—Indices of refraction plotted against composition of orthopyroxenes, showing deviation from linearity. The straight line is drawn for comparison.

are considerably less stable as related to iron-rich olivines than are the magnesium-rich pyroxenes as compared with magnesium olivines. Therefore, when the average Mg/(Mg + Fe) ratio is less than a certain value (some 0.6–0.5), olivine is able to absorb much more iron than pyroxene; at larger average Mg/(Mg + Fe) values, pyroxene will take as much iron as olivine or more. These relations indicate that pyroxenes of intermediate composition should have a relatively greater lattice energy or less molal free energy than that which corresponds to the same mol fractions in ideal mixtures.

One should expect that a relatively great stability in certain portions of a mixed-crystal series would reflect itself qualitively on the mol volume, so that there would be a tendency toward contraction and hence a negative deviation from Vegard's law, which states that the mol volume (or unit cell) is a linear function of the compositions. Again, the refractive indices may indicate such a contraction so that they should show a positive deviation from linearity. The indices of refractions of the clinoenstatite-clinoferrosilite series, measured by Bowen and Schairer (1935, p. 199), plotted against weight percentage are straight lines. This means that plots of n_α and n_γ versus mol fraction will be curves with the convex side up. The same characteristics are shown by the n_α and n_γ indices of the rhombic pyroxenes (fig. 4). It is well known, on the other hand, that the ideal olivine mixed crystals have indices of refraction which vary linearly with mol fraction.

To check the indication of negative deviation of the orthopyroxenes from Vegard's law as suggested by the indices of refraction, X-ray measurements of a series of orthopyroxenes were made. A 5.7-cm.-radius Phillips powder camera and FeK_α radiation were used. The a and b parameters were measured by using the good reflections (060), (650), (12, 0, 0), and (14, 5, 0). The (14, 5, 0) reflection is strong, with an angle θ varying from 65°.55 to 67°.40. The parameters a and b were measured by (060), (650), and (12, 0, 0). Corrections for film shrinkage and rod thickness were made.[3] The Bragg angles of the (060) and (12, 0, 0), as well as the (650) reflections, are approximately the same; thus these reflections should be subjected to the same systematic errors due to the rod thickness and uneven film shrinkage. It is therefore assumed that the ratio between a and b measured from (060), (12, 0, 0), and (650) is nearly correct, even if the absolute values of a and b may be less accurate. By means of this ratio, $[a\ (12, 0, 0)]/[b\ (060)]$, the values of a and b were recalculated so as to fit the good back-reflection (14, 5, 0). The value of FeK_α is taken as 1.93597 Å. The maximum error of the a and b recalculated on the basis of (14, 5, 0) is assumed to be 0.002 Å. Table 5 and figure 5 show the results.

The chemical compositions were first determined by the index of refraction, but the large scattering of the X-ray data soon indicated that index of refraction was insufficient to show the chemical composition of the pyroxenes investigated by X-ray. We, therefore, subsequently analyzed those of our samples which could be prepared for reliable

[3] The half-circumference, $C/2$, of the shrunken film was measured as the distance between the middle point of several corresponding front reflections and the middle point of several corresponding back reflections. The film was supposed to have shrunk evenly. The distances X on the film corresponding to the double deviation angles, 4θ, for the several reflections were measured, and a correction, $\Delta L = r(1 + \cos 2\theta)$, was subtracted. The symbol r is the radius of the rod. The corrected deviation angle was then taken as

$$2\theta = \frac{(X - \Delta L)\,180}{C} \text{ degrees}.$$

TABLE 5

UNIT CELL DIMENSIONS OF RHOMBIC PYROXENES

No.	X	a from (12, 0, 0) (Å)	b from (060) (Å)	a from (14, 5, 0)* (Å)	b from (14, 5, 0)* (Å)
37393	0.911	18.232	8.832	18.225	8.828
344	.720	18.278	8.850	18.281	8.852
37096	.725	18.247	8.816	18.256	8.821
329	.710	18.278	8.875	18.279	8.876
6-82-B	.656	18.286	8.857	18.279	8.847
37859	.590	18.283	8.898	18.292	8.900
27218	.467	18.313	8.922	18.317	8.924
XYZ	0.135	18.395	9.027†	18.408	9.033

* See text.

† Mean b from (060) and (650).

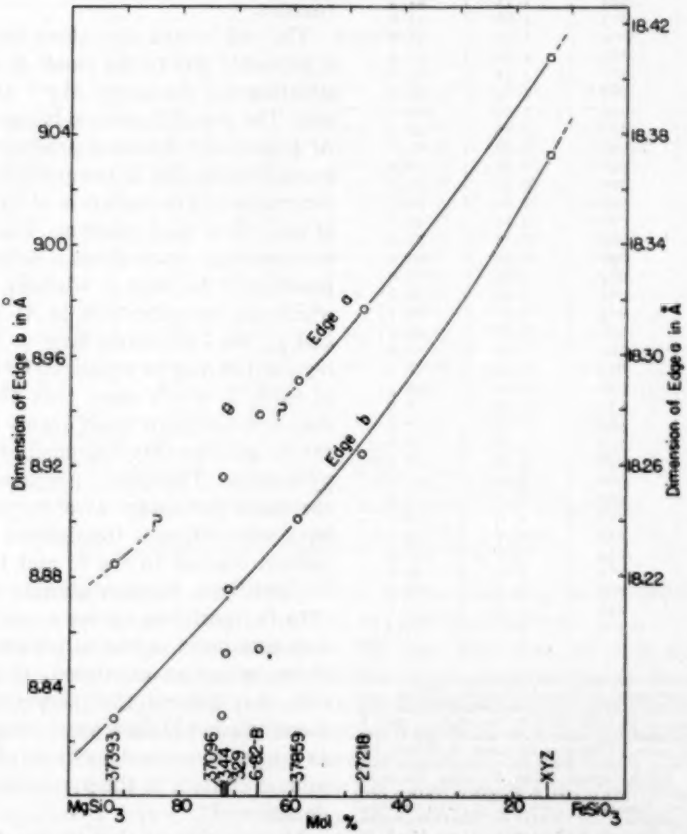

FIG. 5.—The *a* and *b* parameters of the unit cell of the orthopyroxenes. Data are listed in table 5. Specimen numbers are given at the bottom of the figure.

TABLE 6

ALUMINUM CONTENT OF CHEMICALLY ANALYZED ORTHOPYROXENES CALCULATED AS ATOMIC PER CENT Al OF ALL METALS. AND THE DISTRIBUTION OF ALUMINUM BETWEEN THE SILICON AND Mg, Fe POSITIONS OF THE MINERAL

No.*	X_p	Atomic Per Cent Al	Per Cent Al in Si Position
1	0.994	0.00	00.0
A	.911	0.43	100.0
2	.907	0.96	98.8
3	.896	1.75	100.0
4	.888	1.75	62.5
5	.817	4.68	53.8
6	.814	1.57	94.2
7	.809	4.31	53.9
B	.781	3.74	69.8
8	.781	4.13	79.7
9	.758	2.39	100.0
C	.756	7.03	56.9
D	.753	6.66	13.6
10	.740	2.36	50.0
E	.725	6.44	46.2
F	.722	5.61	80.9
G	.720	4.13	44.1
H	.707	6.81	71.2
11	.698	0.54	100.0
12	.681	1.87	85.0
13	.681	1.52	56.6
I	.664	3.71	77.7
J	.656	6.07	59.4
K	.614	7.23	0.0
L	.612	8.74	52.3
M	.580	1.92	95.3
N	.571	10.48	0.0
O	.566	6.83	0.0
P	.554	9.12	42.8
14	.526	4.08	47.9
15	.500	3.19	46.5
R	.467	2.48	35.4
16	.453	2.19	9.1
17	.238	0.86	73.9
18	.234	1.82	36.4
19	.167	0.13	90.0
20	.166	0.36	100.0
S	.135	1.50	0.0
21	0.118	1.12	57.4

* Nos. 1–21 are analyses 1–21 in Poldervaart (1947, p. 168); A, no. 37393, this paper; B, Bushveld (Hess and Phillips, 1938, p. 453, anal. no. 3); C, Labrador (Washington and Merwin, 1923, p. 64, anal. no. 2); D, India (Srirama Rao, 1947, p. 138); E, no. 37096, Greenland, this paper; F, Heinola, Finland (Lokka, 1943, p. 29); G, no. 344, Greenland, this paper; H, Heinola, Finland (Lokka, 1943, p. 29); I, Adirondacks (Buddington, 1939, p. 32); J, no. 6-82-B, Laramie Range, Wyoming, this paper; K, Uganda (Groves, 1935, p. 152); L, Minnesota (Bayley, 1895, p. 5); M, Turku, Finland (Hietanen, 1947, p. 1037, no. 1059); N, India (Rajagopalan, 1946, p. 317); O, India(Rajagopalan, 1946, p. 315); P, Terra del Fuego (Lokka, 1943, p. 32); R, no. 37218, Greenland, this paper; and S, no. XYZ, Greenland, this paper.

chemical analyses. Unfortunately, some of the already X-rayed samples could not be analyzed because of some weathering. The noticeable decrease in the size of the *b* edge of the unit cell given by points 37096 and 6-82-B is accompanied by a high aluminum content in these pyroxenes. Aluminum makes up 6.44 per cent of the total metals in 37096 and 6.07 per cent in 6-82-B (see fig. 5 and tables 6 and 7). Even with the high content of aluminum in the mineral, the index of refraction falls on the curve established by aluminum-poor or -free orthopyroxenes.

The cell contraction along the *b* edge is probably due to the small Al ion substituting for the larger Mg^{++} and Fe^{++} ions. The simultaneous exchange of Si by Al apparently does not produce a large enough expansion to compensate for the contraction. The variation of the *a* edge of the cell is more puzzling. The Al-rich pyroxene no. 37096 shows a definite contraction of the edge *a*, whereas no. 344, which also is rather rich in Al (tables 6 and 7), has a relatively large *a*. These irregularities may be explained by the data in table 6, which show that aluminum does not distribute itself evenly between the Si and the (Mg,Fe) positions in the pyroxenes. Therefore, pyroxenes with the same percentage of aluminum may have very different fractions of the aluminum content in the Si and the (Mg, Fe) positions. Because aluminum in the (Mg,Fe) positions causes a contraction, and aluminum in the Si positions most likely causes an expansion, of the unit cell, it is natural that the unit-cell dimensions should show some irregularities as long as the actual positions of the aluminum content of the pyroxenes are not determined.

It is significant that, even when only the aluminum-poor orthopyroxenes are

considered, the dimensions of the unit cell (or at least the parameters *b* and *a*) versus composition are still curved lines which one should expect if the orthopyroxenes are not ideal mixtures between Mg and Fe silicates.

It is also interesting to note that the aluminum content in orthopyroxenes seems to depend upon the Mg/Fe ratio, as is seen from table 6 and figure 6. A region of composition ($X = 0.8$–0.4) includes the most aluminum-rich orthopyroxenes. In order to get a statistical expression for the validity of the sample and the possibility that new data would fall outside present data, the curve on figure 6 representing the mean plus the tolerance factor (Eisenhart *et al.*, 1947, p. 102) times the standard deviation of each class interval is drawn. The tolerance factor is such that at least 95 per cent of any additional data will fall in the field below the curve 75 per cent of the time, assuming normal distribution of data within the class intervals. The probability of additional data following the same distribution is probably higher, since there should be some definite lattice limitations of the orthopyroxenes to the acceptance of aluminum. It is noticed that nine analyses are plotted in figure 6 that are not included in the statistical treatment. These analyses were rejected because they do not recalculate to the standard formula.

TABLE 7

ANALYSES OF RHOMBIC PYROXENES

Oxide	Specimen No.					
	37393*	37096*	344*	6-82-B*	37218†	XYZ†
SiO_2	56.56	51.45	52.64	50.06	49.63	47.51
TiO_2	0.03	0.16	0.16	0.69	0.37	0.28
Al_2O_3	0.43	5.98	3.82	5.57	2.16	1.20
Fe_2O_3	0.32	1.52	0.16	0.88	0.58	1.97
FeO	5.33	15.12	16.85	19.30	30.69	44.03
MnO	0.08	0.26	0.22	0.32	0.30	0.05
MgO	36.80	24.56	24.54	21.42	15.22	3.99
CaO	0.16	0.00	0.98	1.68	1.16	0.92
H_2O+	0.31	0.34	0.29	0.45	n.d.	n.d.
H_2O-	0.04	0.12	0.05	0.05	n.d.	n.d.
Total	100.06	99.51	99.71	100.42	100.11	99.95
X	0.9109	0.725	0.720	0.656	0.466	0.135
$n_\gamma \pm 0.001$	1.668	1.702	1.701	1.711	1.734	1.770

* Analysis by H. B. Wiik.

No. 37393: $(Ca_{0.012}Mg_{1.870}Fe^{++}_{0.091}Fe^{+++}_{0.033}Mn_{0.009}Ti_{0.003})(Si_{1.910}Al_{0.090})O_{24}$.

No. 37096: $(Ca_{0.009}Mg_{1.317}Fe^{++}_{0.458}Fe^{+++}_{0.041}Mn_{0.007}Ti_{0.013}Al_{0.305})(Si_{1.760}Al_{0.240})O_{24}$.

No. 344: $(Ca_{0.132}Mg_{1.338}Fe^{++}_{0.507}Fe^{+++}_{0.011}Mn_{0.007}Ti_{0.013}Al_{0.150})(Si_{1.882}Al_{0.118})O_{24}$.

No. 6-82-B: $(Ca_{0.163}Mg_{1.222}Fe^{++}_{0.580}Fe^{+++}_{0.098}Mn_{0.010}Ti_{0.037}Al_{0.382})(Si_{1.911}Al_{0.089})O_{24}$.

† Analyses by B. Bruun. Na_2O, K_2O, and P_2O_5 n.d. Minerals calculated to the standard formula $Y_4Z_4O_{24}$:

No. 37218: $(Ca_{0.163}Mg_{1.338}Fe^{++}_{0.903}Fe^{+++}_{0.039}Mn_{0.009}Ti_{0.011}Al_{0.138})(Si_{1.922}Al_{0.078})O_{24}$.

No. XYZ: $(Ca_{0.264}Mg_{0.998}Fe^{++}_{0.925}Fe^{+++}_{0.058}Mn_{0.007}Ti_{0.014}Al_{0.162})(Si_{1.933}Al_{0.067})O_{24}$.

We thus think that our data with reference to the distribution of aluminum in the orthopyroxene series are representative. Aluminum is mainly found in pyroxenes with X ranging from about 0.8 to 0.4. However, this does not necessarily mean that the pyroxene lattice within this range of X is particularly susceptible to accepting aluminum. It may also

mean that the rocks in which Mg-rich or Fe-rich pyroxenes develop have little aluminum and thus the chance for finding aluminum in such pyroxenes is small. On the other hand, if pyroxenes in the interval $X = 0.8–0.4$ are especially susceptible to accepting aluminum in the lattice, then this is only another indication that the orthorhombic pyroxenes do not constitute an ideal mixed crystal series.

SUMMARY

The distribution of Fe and Mg in natural pyroxene-olivine pairs is investigated and compared with the distribution of the same elements in artificial olivine-pyroxene pairs. It is found that the difference in the distribution constant is mainly due to different temperatures of formation of the rocks, these being at lower temperatures than the synthetic samples. It is shown that accu-

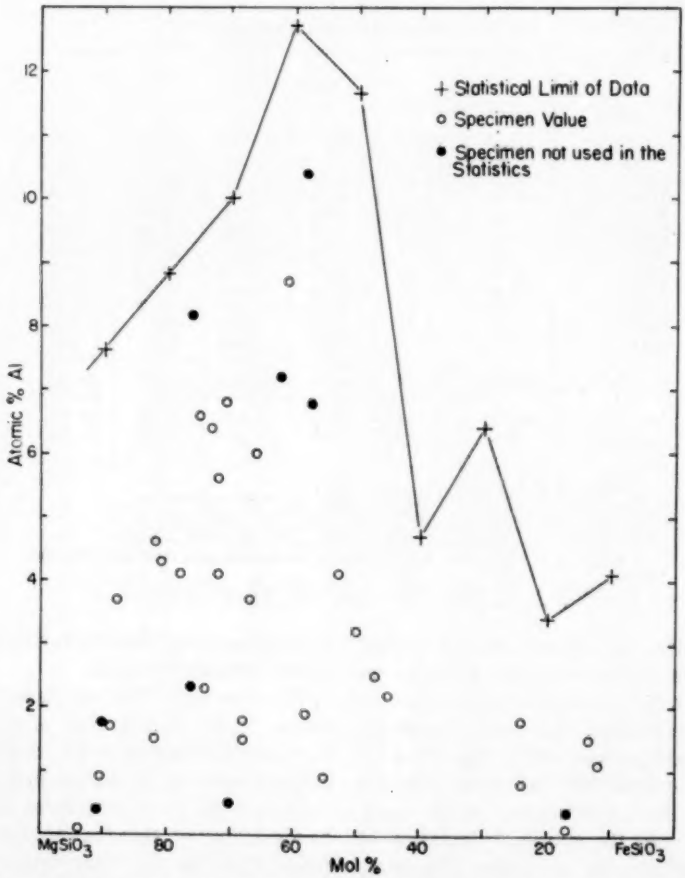

FIG. 6.—Atomic per cent of aluminum in orthopyroxenes plotted against the Mg/(Mg + Fe) ratio. Statistical test for the validity of the distribution is represented by the curve. Aluminum data are listed in table 6, statistical data are listed in table 8.

rately measured entropy data of the system would enable us to calculate temperatures of formation from the distribution of Mg and Fe in olivine-pyroxene pairs in rocks.

The orthorhombic pyroxenes do not represent an ideal mixed-crystal series. This is shown by the Fe-Mg distribution as well as by the index of refraction and the dimension of the unit cell across the series.

considerably. The effect of Al upon the *a* parameter is not clear because some Al-bearing pyroxenes do have normal or even large *a*, while others show contraction. This may be due to the fact that Al can distribute itself in any proportion between the Si and the Mg,Fe positions in the lattice.

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TABLE 8

STATISTICAL DATA FOR THE LIMIT CURVE IN FIGURE 6

Class Interval* (C.I.)	No. of Samples (N)	Standard Deviation (s)	Tolerance Factor† (K)	Sample Mean (M)	$M+Ks$
1.0 –0.81	6	1.87	2.89	2.21	7.61
0.90–0.71	12	1.82	2.46	4.35	8.83
0.80–0.61	13	2.17	2.42	4.75	10.00
0.70–0.51	8	3.05	2.66	4.63	12.74
0.60–0.41	6	2.71	2.89	3.83	11.66
0.50–0.31	3	0.51	4.19	2.62	4.75
0.40–0.21	2	0.69	7.41	1.33	6.44
0.30–0.11	5	0.82	3.09	0.90	3.43
0.20–0.01	3	0.77	4.19	0.88	4.11

* Class interval of 0.19 includes all samples that have $Mg/(Mg + Fe)$ ratios within this composition range.

† The tolerance factor is such that, with a risk of 0.75, at least 95 per cent of additional data will be included within the limit curve. Values of the tolerance factors were taken from Eisenhart, Hastay, and Wallis (1947, p. 102).

The aluminum content in pyroxenes is discussed briefly. It is found that the chance of finding aluminum in rhombic pyroxenes is greater for pyroxenes of intermediate to Mg-rich composition (X = 0.8–0.4). Al does not noticeably affect the index of refraction but makes the *b* parameter in the unit cell contract con-

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SIZE VERSUS SORTING IN SOME CARIBBEAN SEDIMENTS[1]

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ABSTRACT

The relationship between average size (phi median) and sorting (phi percentile deviation) in sediments is explored empirically by means of statistical analysis of data from over twelve hundred individual samples taken from Miocene and Oligocene horizons in southern Trinidad, the Eocene Scotland series from Barbados, and the Pliocene Superficial Gravels of British and Dutch Guiana. The association between size and sorting is estimated by means of the correlation coefficient, and the relationship is reduced to linear equations of the form $PD_\phi = bMd_\phi + a$. Deviations from the trends are measured in terms of the standard error of estimate (Sy). On the basis of the statistics it is shown that, in water-deposited sediments, size varies with sorting over the range 0ϕ (1.00 mm.) to 6ϕ (0.0156 mm.) in median diameter and 0–4.5ϕ in phi percentile deviation. Best-sorted sediments ($PD_\phi < 1\phi$) possess median diameters around 2.5ϕ (0.177 mm.), and sorting becomes poorer with both increasing and decreasing size from the fine-sand range (2–3ϕ). Departures from the trend of over 1 Sy unit indicate that in finer-grained sediments (clays and fine silts) flocculation had taken place during grain-size analysis; whereas in coarser-grained sediments (very fine sands and coarse silts) the departures imply events of geological significance which interfered with normal sedimentation.

INTRODUCTION

The treatment of measured data resulting from investigations into the petrography of sediments has been systematized by Krumbein (1934*a*, *b*, 1936*a*, *b*, 1938, 1939, 1940; Krumbein and Aberdeen, 1937; Krumbein and Pettijohn, 1938); the principal emphasis was perforce devoted in the early stages to the reduction of grain-size distributions to a few representative statistics. Subsequently, Krumbein illustrated how such statistics could be further used in interrelating size with other measured characteristics of grains (Krumbein and Pettijohn, 1938) and in following changes in size within a depositional unit (Krumbein and Aberdeen, 1937). One of the aims of Krumbein's work was to study the interrelationships of the statistics among themselves (1940) and to elucidate this interrelationship in terms of sedimentation. It is important to realize that if, as the earlier pioneers of sedimentary petrography believed, the variation of the several parameters—size, shape, etc.—in sediments is infinite, then the problem of studying measured characteristics of sedimentary rocks becomes insuperable. Krynine, however, approached the problems of sedimentary petrography on the implied, if not stated, basis that, although it is theoretically possible for this infinite variation to occur in, for example, textural characters and mineral composition, in practice it appears that there are very definite limitations and that within single rock groups there is a clear tendency for a preferred texture to be associated with a preferred composition (1943). If Krynine's approach is correct, then the solution to most problems of sedimentary petrography not only becomes possible of achievement but also can be accomplished with a relatively few representative samples when once the interrelationships between the significant variables of sedimentary rocks have been established. The present investigation is aimed at the elucidation of the interrelationship between size and sorting in sedimentary rocks.

A relationship between size and sorting of a sediment may be deduced on a priori grounds from the behavior of a

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transporting medium. Indeed, as current velocity ultimately determines both capacity and competence, it will also exercise a definite size selection on the material available for transportation. The most obvious relationship is that between average current velocity and the most abundant particle diameter of a sediment—the mode. In addition to the mode, however, if the exact nature of the transport is restricted to either suspension, saltation, or traction, then the range of sizes is also closely determined by the current velocity. It appears from this that size and sorting are controlled by similar factors and hence that there should be a traceable, if not direct, relationship between them.

On this premise the author has for some years been compiling grain-size analyses both from published work and from his own investigations, and it is clear that the relationship can be traced fairly simply for water- and air-borne sediments; whereas, as would be expected, gravitational and glacial deposits do not follow the same pattern. This discussion is limited to the author's results in Trinidad, British West Indies, and contiguous areas and is a preliminary description of the method of approach and results obtained by empirical testing of the relationship between size and sorting in water-borne sediments.

Analytical Procedure

Two statistics form the basic data upon which the subsequent computations are performed; they comprise the median diameter and percentile deviation, both measured in terms of the phi scale (Krumbein, 1936). The phi median (Md_ϕ) is obtained by reading the phi diameter at the 50 per cent point in the size distribution. It is obtained by direct reading from the cumulative curve and is a commonly used "average" diameter based upon geometrical position in the center of the distribution. It is not affected by the extremes of the range and is therefore particularly suited to "open-ended" distributions. The phi medians of the sediments used for this investigation approach the modal values more closely than does either the arithmetic or the geometric mean (fig. 1) and hence are a

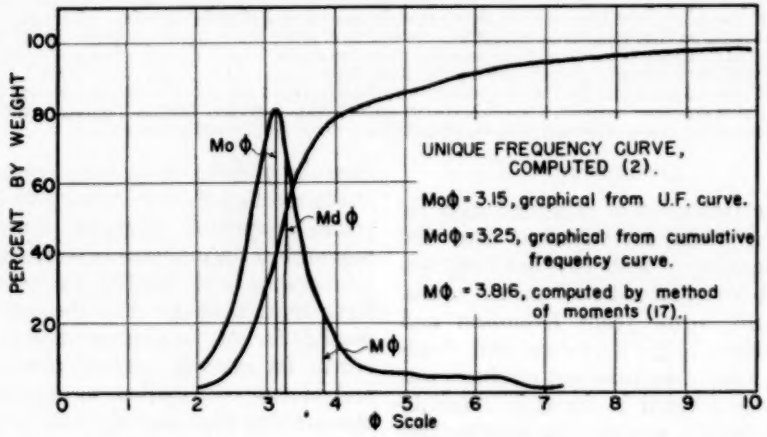

Fig. 1.—Cumulative and unique frequency curve of typical Miocene sand, southern Trinidad

closer approximation to this genetically important diameter than is any other average. It is, of course, much simpler to derive than the mode which can be satisfactorily obtained only from the unique frequency curve (Krumbein, 1938; Brotherhood and Griffiths, 1947).

The phi percentile deviation (PD_ϕ) is analogous to the more orthodox phi quartile deviation (Krumbein, 1936*b*), but the phi quartile diameters are replaced by the phi diameters at the 10 and 90 percentiles. The formula for derivation becomes

$$PD_\phi = \frac{P_{90\phi} - P_{10\phi}}{2}. \qquad (1)$$

This statistic is used as a measure of sorting and represents half the spread between the ninetieth and the tenth percentile diameter values. It has several advantages over the phi quartile deviation while still retaining its ease of derivation. The percentile deviation encompasses some 80 per cent of the distribution, as compared with 50 per cent for the quartile deviation. It is equally applicable to open-ended curves and, as would be expected, is more sensitive to fluctuations in the fine ($P_{90\phi}$) and coarse ($P_{10\phi}$) ends of the distribution. It has, of course, a greater range in absolute value than the quartile deviation has, and this is also some advantage. It is obtained by reading the phi diameters at the tenth and ninetieth percentiles, respectively.

It is important to emphasize here that the three diameter values—median, 10, and 90 percentiles—are mutually independent. Variation in one does not necessitate or imply variation in either of the others. Thus at the outset the two statistics—median (average size) and percentile deviation (sorting)—are both mathematically and petrographically independent. Any relationship established between these two measures is therefore not implicit in the manipulations. The succeeding mathematical treatment is conventional and uses the standard symbols X and Y for the two variables. It is necessary, therefore, to decide which statistic shall be represented by the independent variable X and which by the dependent variable Y. It seems clear that the more accurately determined variable is Md_ϕ based on a single reading in the steepest part of most cumulative curves, and Md_ϕ is, therefore, designated X; by the same reasoning PD_ϕ is dependent on two readings in the asymptotic parts of the curves and hence is least accurate and becomes the dependent variable Y. It is interesting to note that this is in agreement with their petrographic significance, since the modal value, of which the median is a close approximation in this case, is much more precisely circumscribed than is the range by the current velocity.

The phi medians and phi percentile deviations of over one thousand grain-size distributions of Tertiary sediments in Trinidad and contiguous areas were compiled, and these form the basic data upon which the subsequent manipulations were performed. In order to evaluate empirically the relationship between size and sorting, scatter diagrams were constructed of average size (Md_ϕ) versus sorting (PD_ϕ), and a discernible trend was displayed in most cases (figs. 2–12). The measure of correlation between size and sorting can be usefully expressed by the correlation coefficient. The raw-data formula was used (Rider, 1947, p. 48):

$$r = \frac{\Sigma XY - [(\Sigma X \cdot \Sigma Y)/N]}{\sqrt{\Sigma X^2 - (\Sigma X)^2/N}\sqrt{\Sigma Y^2 - (\Sigma Y)^2/N}}, \qquad (2)$$

where r is the correlation coefficient and Σ is the sign for summation, X and Y are Md_ϕ and PD_ϕ, respectively, and N is the number of samples.

In this computation random selections of 15–100 samples were made from various sets of data, and, using r and the number of samples, the statistical significance of the correlation coefficient may be directly tested from Fisher's table (1948, p. 209). Most of the correlation coefficients computed during the present work proved to be statistically significant (P much less than 0.01 and in many cases less than 0.001). Literally, this test is carried out by assuming that the "parent-population" of Md_ϕ and PD_ϕ values is uncorrelated and then computing how likely an r of the value obtained from the sample of Md_ϕ and PD_ϕ represented by the data is to arise from this uncorrelated population. In the present work this likelihood (P) was found to be less than 1 in 100 ($P < 0.01$) or less than 1 in 1,000 ($P < 0.001$), and hence it is assumed that a correlation exists in the population from which the sample tested was drawn. Those samples which failed this test are discussed where relevant in the text.

The presence of association once established, the next step is to compute the relationship between the variables. This was accomplished by the "method of least squares" (Mills, 1938; Fisher, 1948) and the line of "best fit" was computed by means of the normal equations

$$\left.\begin{aligned} \Sigma Y &= Na + b\Sigma X \,, \\ \Sigma XY &= a\Sigma X + b\Sigma X^2 \,, \end{aligned}\right\} \qquad (3)$$

where the notation is similar to that in equation (2) and a and b are the unknown coefficients to be determined.

The spread of the values around this line of best fit was measured by the standard error of estimate (Sy), and this was computed from the following formula (Mills, 1938):

$$(Sy)^2 = \frac{\Sigma(Y^2) - a\Sigma(Y) - b\Sigma(XY)}{N} . \qquad (4)$$

The notation is the same as for equations (2) and (3).

The line of best fit and the estimated spread ($\pm Sy$) are plotted on the scatter diagrams and used as a standard of reference for investigating unusual departures from the linear relationship. On the basis of the implied assumptions that the relationship between size and sorting is linear, the trends may all be expressed in the following form:

$$PD_\phi = bMd_\phi + a \,. \qquad (5)$$

Description of the Results

In order to shorten the description, it will be sufficient to outline one example in detail and trace the steps followed and the assumptions involved. Where subsequent treatments are identical, only the computed trends need be dealt with during the description of the results. In some cases, however, deviations from the trends become important, and the distribution of the observations over the scatter diagram is then given in full, but the mathematical steps involved are only briefly stated.

Application of the Procedure

Well A in Forest Reserve Field, southwestern Trinidad, was cored through part of the Miocene sequence of sediments. Sixty-five samples from these cores were subjected to grain-size analysis, and the phi median and 10 and 90 percentile diameter values were read from the cumulative curves. The phi percentile deviations were obtained by substitution in formula (1).

The values of Md_ϕ and PD_ϕ were then

plotted in a scatter diagram with Md_ϕ as abscissa (X) and PD_ϕ as ordinate (Y). The resulting trend is shown in figure 2. A sample of 25 pairs of values (Md_ϕ and PD_ϕ) was randomly selected from the 65 for purposes of computation. Randomness was insured by using standard tables of random numbers (Snedecor, 1946; Fisher and Yates, 1948). The 25 pairs of values were then tabulated, and the necessary sums (ΣX, ΣY), sums of squares (ΣX^2, ΣY^2), and sum of products (ΣXY) of the values were obtained on an automatic calculator. From these values the correlation coefficient r, the least-squares trend, and the standard error of estimate were computed by using equations (2), (3), and (4), respectively. In this case the sample of 25 paired values gave a correlation coefficient, $r = 0.9408$. Assuming that the set of 25 values is actually taken from an infinite number of such pairs of values (population) in which there is no correlation between Md_ϕ (X) and PD_ϕ (Y), the chance of obtaining an r of this magnitude is much less than 1 in 1,000, i.e., $P < 0.001$ (Fisher, 1948). In this sense, therefore, it appears that the measure of association between size (Md_ϕ) and sorting (PD_ϕ) is statistically significant.

Computing the line of best fit by the method of least squares (eq. [3]), the following relationship results: $PD_\phi = 0.6190Md_\phi - 0.3510$; and this is plotted in the scatter diagram (fig. 2). As a measure of the scatter of the data around this line, the standard error of estimate computed from equation (4) gives $Sy = 0.27$; statistically, this implies that if the values belong to normal distributions, some 68 per cent of all the values should lie around the line of best fit and within 1 Sy unit on either side of it. Parallel lines are inserted at 1 Sy unit on either side of the line of best fit (see fig. 2), and it can be seen that, of these 25 pairs of values, more than two-thirds and nearer three-quarters fall within the specified limits. From the entire 65 values, however, far more than one-third fall outside these Sy limits.

A second set of 20 pairs was selected from the original data after the previous 25 had been eliminated from consideration. The correlation coefficient derived from this second set of values gives $r = 0.9085$, and the statistical significance is equally high ($P < 0.001$). The least-squares trend is $PD_\phi = 0.806Md_\phi - 0.954$, and the standard error of estimate is $Sy = 0.49$. It appears from this that the Sy value calculated from the first set of pairs is fortuitously low.

From this one example it can be seen that the sorting varies with size, and from the computed trends it is also clear that, as Md_ϕ increases, PD_ϕ also increases. In millimeter units this signifies that sorting becomes poorer as the sediments become finer over the size and sorting limits represented by these values, i.e., between the Wentworth grades of fine sand and coarse silt (i.e., $Md_\phi = 2.0$–6.0). It appears convenient at this point to emphasize some of the basic assumptions involved in the analytical procedure which leads to this conclusion. It is not possible to establish these assumptions as valid at this stage, but at least a brief discussion will reveal the steps in the analysis necessary for their ultimate establishment. Three principal questions arise: Is it correct to assume that the trend is linear? What is the nature of the sampling error involved? What is the significance of the standard error of estimate as a measure of the spread of the values around the trend line?

Examination of the data in the scatter diagram clearly indicates that, if only

those points with medians less than 4.5ϕ and percentile deviations less than 3.0ϕ are considered, the trend, if linear, would be much steeper than either of those computed. Such a trend is confirmed by a single value from the 65 beyond the limits of the scatter diagram (Md_ϕ = 5.15ϕ; PD_ϕ = 4.60ϕ). The decrease in the slope of the line is, therefore, primarily due to those points lying between Md_ϕ = 5–8ϕ and PD_ϕ = 3–5ϕ. Several alternative explanations exist for such occurrences. The points beyond Md_ϕ = 5ϕ may be in error;[2] the trend is actually curvilinear; or the trend in the two ranges may be better expressed by two straight lines. Without a much larger number of points in the region Md_ϕ = 4–8ϕ, it is not possible to decide between these alternatives, and, hence, for the present it is convenient to assume a linear trend as a first approach.

The second question concerning the variation in trend line due to sampling error can be tested empirically and certain practical limits on the number of samples used for computation thereby established. Sixty-nine subsurface samples from well B, Forest Reserve Field, southwestern Trinidad, were analyzed, and the resulting Md_ϕ and PD_ϕ values were plotted in a scatter diagram (fig. 3). Successive sampling in sets of 15, 20, 25, 30, and 69 pairs of values was performed and the statistics computed in turn. The

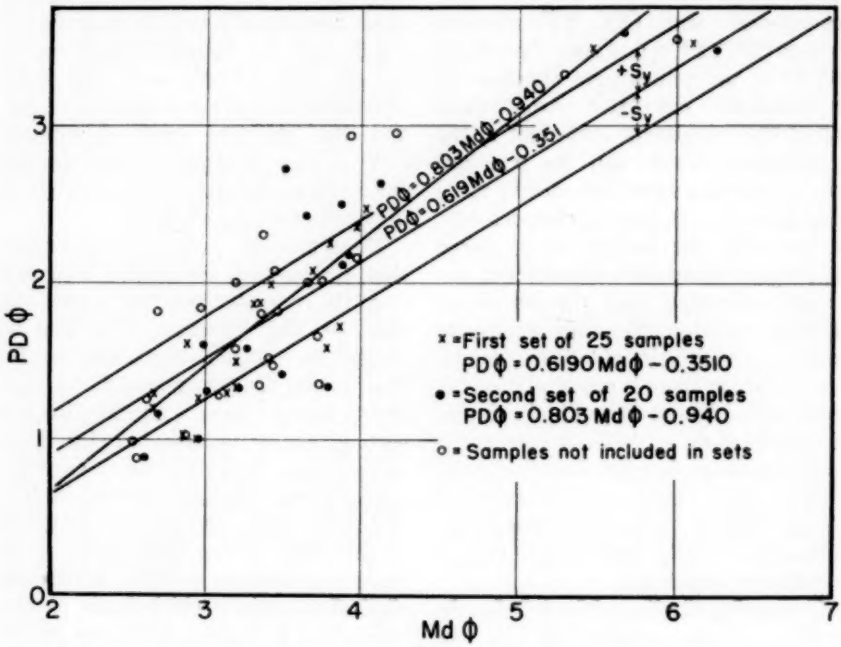

FIG. 2.—Size versus sorting relationship, well A, Miocene, southwestern Trinidad

[2] A simple assumption would be to consider these points as representative of clayey sediments which had suffered flocculation. This possibility is discussed more fully on p. 222.

results are summarized in table 1. It may be seen that the correlation coefficient r varies between 0.4054 and 0.5077, a small range compared with the change in the number of samples. In the case of the set of 15, however, a value of r (0.4524) would arise because of errors of random sampling from an uncorrelated population about 1 in 10. This can hardly be considered significant in a statistical sense and would certainly not be evidence of a valid association in a geological sense. With a set of 20 pairs of values, the position improves; but such a correlation as that listed in the table could arise about once in 20 times from a set of 20 pairs drawn from an uncorrelated population. There is little real

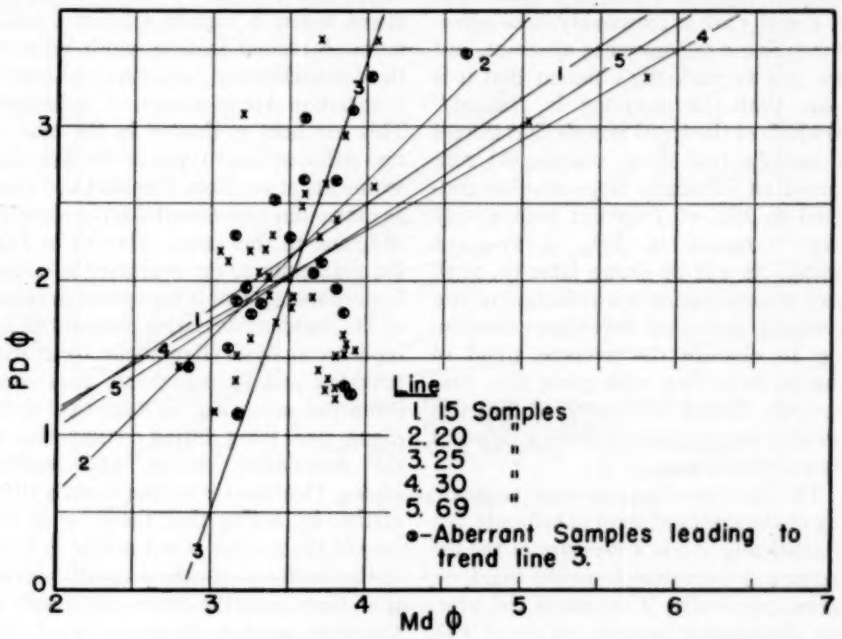

FIG. 3.—Size versus sorting relationship, well B, Miocene, southwestern Trinidad

TABLE 1

SIZE VERSUS SORTING RELATIONSHIP IN 69 SUBSURFACE CORES FROM WELL B, MIOCENE, TRINIDAD

No. of Samples	Least-Squares Trend	Standard Error of Estimate	Correlation Coefficient	P
15	$PD_\phi = 0.6806 Md_\phi - 0.2263$	0.81	0.4524	<0.10
20	$PD_\phi = 1.0139 Md_\phi - 1.4072$	0.67	0.5077	<0.05
25	$PD_\phi = 2.9214 Md_\phi - 8.2689$	0.99	0.4320	<0.05
30	$PD_\phi = 0.5859 Md_\phi - 0.0177$	0.46	0.4054	<0.05
69	$PD_\phi = 0.6584 Md_\phi - 0.2745$	0.59	0.4616	<0.001

change up to 30 samples; but, with 69 pairs and a value of $r = 0.4616$, such a value would arise much less than 1 in 1,000 from an uncorrelated population, and hence the association is statistically valid.

Examination of the trend lines in figure 3 indicates the amount of variation to be expected solely from random sets of samples from the same set of data; lines *1*, *4*, and *5* are in reasonably close agreement; line *2* is somewhat aberrant, but line *3* is so markedly different that it is clear that the sampling is "biased." Analysis of the trend reveals that the set of samples (see fig. 3, samples ⊗) contained an unusually large number confined to Md_ϕ of 3–4ϕ but with a very large variation in PD_ϕ (1.125–4.03ϕ units). As will be shown later (p. 223), such a combination is a reflection of two events of geological importance combining to obscure the average trend of change in sorting with grain size. Statistically it may be considered that this peculiar set of values is from a different parent-population.

The third question concerns the meaning of the standard error of estimate, *Sy*. Statistically, *Sy* is a measure of the departure of the values from the trend, or, more specifically, it measures the average discrepancy between values of PD_ϕ computed from the least-squares trend equation and those actually observed. One of the more obvious reasons for such variation is due to "experimental error," wherein this error includes sampling error between individual samples and arises from choice of sedimentation units (Apfel, 1938; Otto, 1938). This has not been separately evaluated in the present case but would principally affect laminated sediments, in which, owing to the nature of the samples, many sedimentation units would be included in one sample. This would imply that "silty" sediments with Md_ϕ between 4 and 6ϕ would be the most likely types to show large variation. As such an expectation is not borne out in any of the sets of samples tested, it is apparent that the individual sampling error is not very important. It will, however, mean that from this cause, i.e., mixed sedimentation units, the value of *Sy* can never be reduced below a certain unknown minimum. A second feature which adds to the comprehensive "experimental error" is variation due to analytical technique. This has been evaluated in the case of two different sand types in the Miocene sediments of southern Trinidad and gave as maximum experimental error 0.07ϕ in Md_ϕ due to this cause, whereas in PD_ϕ the maximum so far evaluated is 0.24ϕ. Experimental error is here given in terms of the standard deviation determined by replicate analysis (Krumbein, 1934; Rittenhouse and Connaughton, 1944) and comprises errors due to analytical technique, smoothing during construction of the cumulative curves, and reading errors. This can be broken down a little further by noting that the error in the case of the median is not nearly so large as that in the case of the percentile deviation. Here again the errors can be split in the latter case into those associated with $P_{10\phi}$ and those associated with $P_{90\phi}$. From experience with the Trinidad Miocene sediments it can be stated that the errors associated with $P_{10\phi}$ are not very different from those associated with the median, whereas $P_{90\phi}$, as can be seen from figure 1, is very greatly affected by very small differences in smoothing and reading in the flat part of the curve. This alone probably accounts for the greater part of the experimental error in PD_ϕ and, in turn, therefore, with the variation in *Sy*. It seems clear from this discussion

that *Sy* will always be large in sediments with open ends and particularly where the sorting over the whole distribution is poor when compared with the central sorting.

Statistically, *Sy* can be precisely circumscribed only in the case of "normal" distributions, and in such cases it can be shown that 68 per cent of the samples should fall within ± 1 *Sy* unit from the trend lines, 95.4 per cent of the samples within ± 2 *Sy* units from the trend, and only 3 in 1,000 outside the limits of ± 3 *Sy* units from the trend. In the case of well L (fig. 5) these prerequisites are not fulfilled, principally because the distributions of both Md_ϕ and PD_ϕ, but chiefly the former, are not "normal." This is one of the reasons why the strict mathematical treatment, using, say, 2 *Sy* units, cannot be adopted in the present analysis.

THE MIOCENE SEDIMENTS OF SOUTHERN TRINIDAD

WELL SAMPLES FROM SOUTHWESTERN TRINIDAD

Further sets of surface and subsurface samples from the same and different areas of Miocene sediments in southern Trinidad maintain a similar relationship between average size (Md_ϕ) and sorting (PD_ϕ). The generalized relationship always indicates that sorting becomes poorer with decrease in grain size. Some results based on several sets of samples are illustrated in figure 4, and the statistical data are summarized in table 2. The trends, however, indicate that the rate of change varies widely. Two effects can be partially isolated; first, an overall change in average size of a set of samples shifts the trend line across the graph but maintains approximately the

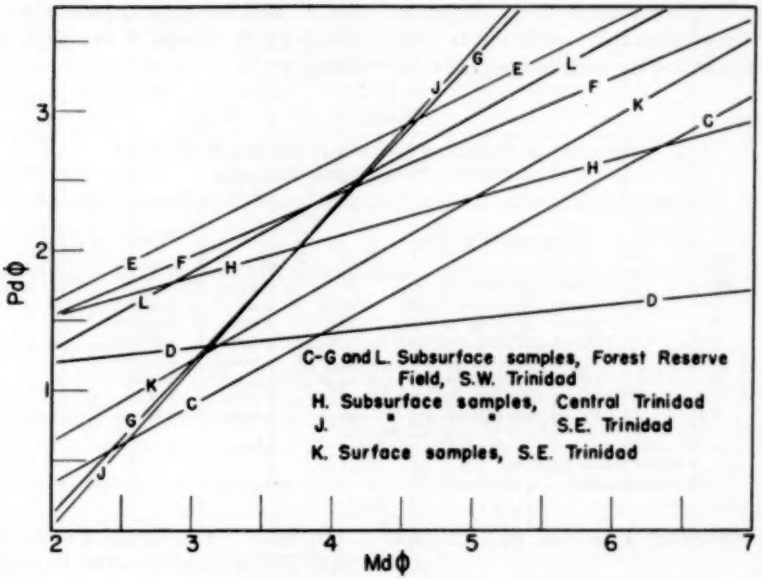

FIG. 4.—Size versus sorting relationship, Miocene, southern Trinidad

same slope. This would account, for example, for the changes in trends *C*, *E*, *F*, and *K*. Second, a change in slope is generally due to "bias" in the sampling, a larger number of sands giving a steeper trend than a set of samples with many clays. This accounts for the divergence of trends *G* and *J*. Trends *D* and *H* are, as can be seen from the table, not statistically significant and could represent sets of samples from uncorrelated populations; it appears most likely, however, that these aberrant trends result from experimental error and, in the case of *D*, that, on the basis of 531 samples, such a value of r would arise much less than 1 in 1,000 by chance from an uncorrelated population. It therefore appears valid to deduce that in the Miocene sediments of southern Trinidad the percentile sorting becomes poorer with decrease in average grain size. The relationship computed as a trend line gives the following equation: $PD_\phi = 0.5574Md_\phi + 0.1754$; $Sy = 0.77$. This trend is inserted in figure 4, trend line *L*, and the general parallelism with trends *C*, *E*, *F*, and *K* is clearly established.

TABLE 2

SUMMARY OF TRENDS IN SIZE VERSUS SORTING IN MIOCENE SEDIMENTS OF SOUTHERN TRINIDAD

Well	Least-Squares Trend	*Sy*	*r*	*P*	No. of Samples
C	$PD_\phi = 0.5567Md_\phi - 0.7709$	0.47	0.5220	<0.01	29
D	$PD_\phi = 0.1034Md_\phi + 0.9905$	0.50	0.0405	>0.10*	36
E	$PD_\phi = 0.5019Md_\phi - 0.6296$	0.70	0.7423	<0.001	50
F	$PD_\phi = 0.4283Md_\phi - 0.6776$	0.51	0.9207	<0.001	50
G	$PD_\phi = 1.0931Md_\phi - 2.0829$	0.47	0.6009	<0.001	84
H	$PD_\phi = 0.2812Md_\phi + 0.9688$	0.48	0.3494	>0.05*	31
J	$PD_\phi = 1.1440Md_\phi - 2.2752$	0.48	0.6566	<0.001	43
K†	$PD_\phi = 0.5797Md_\phi - 0.5311$	0.51	0.6264	<0.001	37

* Not statistically significant.
† Surface samples.

from sampling bias (see pp. 222 and 227).

WELL L FROM SOUTHWESTERN TRINIDAD

The complexity of the variation can be illustrated by a study of a set of 531 samples from well L, Forest Reserve Field in southwestern Trinidad. This well was continuously cored from the surface (approximately 500 feet) to 5,500 feet, a stratigraphic range of some 5,000 feet. The scatter diagram of size (Md_ϕ) versus percentile sorting (PD_ϕ) is shown in figure 5. The correlation coefficient for this set gives a value of $r = 0.7585$; testing for statistical significance indicates

In figure 5 the standard error of estimate (*Sy*) is inserted as two lines parallel to the main trend given above and 1 *Sy* unit away in each case. As a first approximation, it is assumed that values outside ± 1 *Sy* unit are sufficiently different to demand explanation, whereas within 1 *Sy* unit of the trend the variation is due essentially to individual sampling and experimental errors (operational errors, reading errors, etc.). The samples which fall outside the unit *Sy* band fall into four areas, two below the band and two above. It will, however, be more convenient for our present purpose to consider these four groups in order of

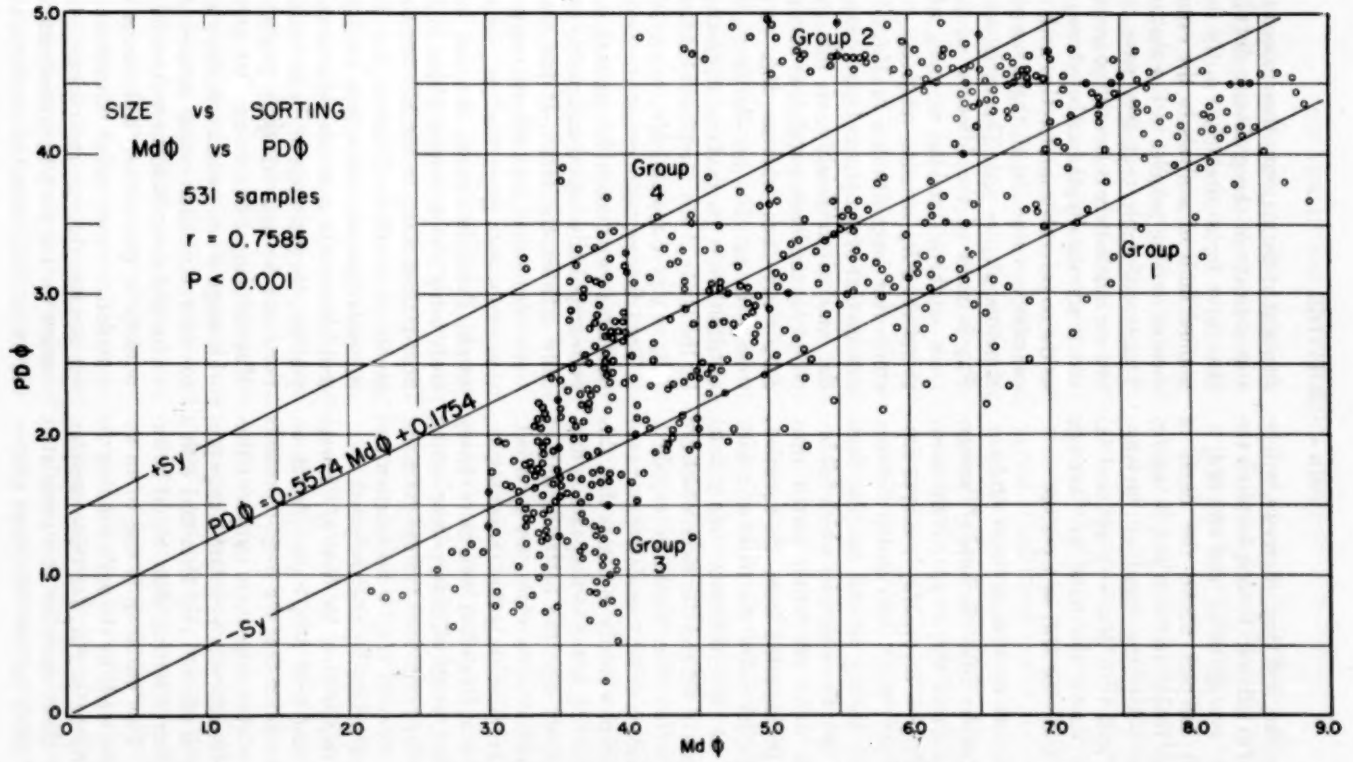

FIG. 5.—Size versus sorting relationship, well L, Miocene, southwestern Trinidad

average size, and they, therefore, will be grouped as follows: (1) samples below the band of "average trend" and with $Md_\phi > 5\phi$; (2) samples above the band of "average trend" and with $Md_\phi > 4\phi$; (3) samples below the band of "average trend" and with $Md_\phi = 3\text{–}4\phi$; and (4) samples above the band of "average trend" and with $Md_\phi = 3.5\text{–}4.5\phi$.

DEVIATIONS FROM THE TREND IN WELL L

1. Samples below the band of "average trend" and with $Md_\phi > 5\phi$.—If the linear trend $PD_\phi = 0.5574Md_\phi + 0.1754$ represents the "true" relationship between size and sorting in the samples from well L, then those samples which fall below the trend are better sorted than would be expected from the equation. The samples below the trend and with $Md_\phi > 5\phi$ are therefore fine-grained sediments of silt and clay sizes, which are better sorted than would be expected. Examination of the cumulative curves of these samples clearly indicates that this better sorting is due to flocculation during analysis, and the better sorting of these samples is therefore only apparent. It appears most likely that these samples, if the size analysis had been more closely controlled, would all show poorer sorting and perhaps even finer average grain size (larger value of Md_ϕ) if flocculation had not occurred during the analysis.

2. Samples above the band of "average trend" and with $Md_\phi > 4\phi$.—Again on the assumption that the trend represents a true relationship, these samples represent sediments which contain a large proportion of silt and clay sizes and with a much poorer sorting than would be expected. This variation is due to an assumption made by the author during the computation of PD_ϕ for these samples. In all these samples the cumulative curves never intersected the 90 percentile, and in the majority they passed the 10ϕ ordinate of the grain-size distribution curve (approximately 1 μ, the fine-grained limit of analysis in the experiments) running parallel to the abscissa. This would imply that they were continuing indefinitely below the 90 percentile. Pragmatically, the author chose 14ϕ as the 90 percentile intersection with the cumulative curve. In all these samples, therefore, $P_{90\phi} = 14\phi$. The variation in $P_{10\phi}$ in most of the Miocene samples from southern Trinidad is small, and, therefore, PD_ϕ remains relatively constant over large changes in Md_ϕ. The distribution of samples in figure 5 shows the marked artificiality of this procedure by the linear parallelism between $PD_\phi = 4.5\text{–}5.0\phi$, whereas Md_ϕ varied from 4 to 7ϕ. The peculiarity of this distribution is particularly emphasized by the rarity of any samples in the range $PD_\phi = 3.75\text{–}4.25\phi$ and $Md_\phi = 5.5\text{–}7\phi$.

The following argument can be used to develop a correction which appears to be petrographically logical and mathematically satisfies the trend. It has many times been stated that mineral composition varies with grain size (see, for example, Pettijohn, 1949), the clay minerals being the dominant phase in the finer-grained sizes. In the Miocene sediments of southern Trinidad the clay minerals comprise illite and kaolinite; and hence, with a known mineral composition, the grain size of the finest fractions can be approximately predicted. Marshall (1931) has shown that, over a wide range of ionic saturation, clays tend to show a median value around 12–12.5ϕ, and these Miocene sediments are unlikely to prove exceptions; hence the cumulative curves which run parallel to the abscissa through 10ϕ probably turn toward the 100 per cent ordinate around 12–12.5ϕ. If this assertion is correct, then

it means that the $P_{90\phi}$ for the samples in this group are in error some 1.5–2ϕ units, and hence PD_ϕ is in error 0.75–1.00ϕ units. Applying this as a "blanket" correction to the samples herein discussed immediately brings them within the belt of average trend. Hence it appears that the majority of these samples are unlikely to be exceptions to the rule expressed in the formula given above, after due allowance is made for experimental error and the author's incorrect initial assumption as to the value of $P_{90\phi}$.

3. Samples below the band of "average trend" and with $Md_\phi = 3$–4ϕ.—These samples are better sorted than would be expected from the formula, but, unlike case 1 above, the sediments are very fine sands and not clays and silts. Clearly, therefore, the same explanation is not sufficient to account for their departure from the trend.

Detailed examination of this group of sediments shows that, of the total 42 samples, 20, or some 50 per cent, are from a single 250-foot sand body and the remainder are stratigraphically widely distributed. The 250-foot sand body is known to be exceptionally uniform on the basis of petrographical characteristics and to extend over a considerable area in which these characteristics are relatively constant. The electrical log pattern of this sand is singularly consistent over the area, as would be expected from the petrographic uniformity, and the sand body is used as a marker horizon on this account. It has been called a "sheet" sand because it is the nearest approach to this type of sand body in the Miocene sediments of southern Trinidad. It very often occurs immediately below an unconformity, and, where absent, it is suspected that it was cut out during the period of erosion represented by the unconformity, which, in turn, indicates the biggest single break during Miocene sedimentation in southern Trinidad.

On the basis of this evidence, one may deduce that this sand represents the top sand of a Miocene sedimentary cycle and, furthermore, that the duration of the period of sedimentation indicated by this exceptionally uniform sediment is a period of stillstand, one of the few well-marked such periods in the Miocene sedimentational history of the area. It was, therefore, a period of reworking and relative "tectonic" quiescence which led to an extremely well-sorted sediment, as indicated by its grain-size distribution. The period of stillstand was followed by a relatively violent renewal of sedimentation and its accompanying "tectonism," and hence this is a well-defined unit in the sequential history of Miocene sedimentation in Trinidad.

By extending this reconstruction, one can suggest that wherever such sands are found in the Miocene sediments, a period of stillstand and reworking occurred. By the argument given above, no other period is so well marked in the historical sequence, and hence the other samples of this group are more widely distributed stratigraphically (and geographically, as can be shown from other areas sampled). Hence the divergence of this group of samples from the average trend is attributed to relatively unusual conditions of sedimentation during the Miocene sedimentation in southern Trinidad.

4. Samples above the band of "average trend" and with $Md_\phi = 3.5$–4.5ϕ.—These samples are more poorly sorted than would be expected from the average trend, and they represent very fine sands and coarse silts in size. In general, they are also more mixed, containing a relatively larger amount of coarser and finer

sizes than do the average Miocene sediments. This, of course, is the factual explanation of their position on the scatter diagram. The geological significance of this departure from the trend is also of some considerable significance in terms of sedimentational history.

The average Miocene sediment of southern Trinidad is a graywacke-type sediment (Krynine, 1948) and is relatively consistent in grain-size distribution. The sediments have been characterized as deltaic in environmental type (Griffiths, in press) and possibly represent the subsea deposits of the Miocene equivalent of the present-day Orinoco. Their size and mineral composition are typical, and, owing to detailed sampling, the petrographic characteristics of these sediments have been thoroughly evaluated. Sediments which belong in the present group contain elements not typical of the average deltaic type. They contain an admixture of minerals and sizes which differ from the true delta type. The accessory (heavy) minerals suggest that these elements are derived from a local, possibly Oligocene and older, source within the area of southern Trinidad. By a somewhat less direct argument it can be shown that the coarser sizes are also locally derived.

The sedimentational characters of delta deposits of large rivers are characteristic of transported materials in which the transport has had around 1,000 miles in which to impress its effects on the materials. The very thoroughly investigated Mississippi delta sediments are an outstanding example (Krumbein and Aberdeen, 1937; Russell and Russell, 1939). Such sediments are usually representative of the suspension load of a large river and, as such, have a closely circumscribed upper size limit. Such rivers characteristically transport materials which vary from fine sand to clay, and the modal size is generally silt (Krumbein and Aberdeen, 1937; Russell and Russell, 1939; Tercier, 1939). The Miocene sediments of southern Trinidad fulfil these requirements almost exactly, although the amount of clay is generally in excess of that of the typical Mississippi deposit. This has been attributed by the writer to the fact that the Trinidad deposits represent the subsea facies of the Miocene river delta, a suggestion supported by the well-marked and oft-repeated marine incursions throughout the Miocene succession. Hence on the basis of this characteristic sedimentational history it is very difficult to understand how coarser materials can be added to the deposits by way of the main river; the most obvious alternative, confirmed by other features, is the local source of such material. The major difficulty lies in explaining how an area without marked relief can be the locus of deposition and a source area at one and the same time. In the case of the Trinidad deposits there is a simple solution to this enigma—the source of the abnormal material lies in the outbursts of local mud volcanoes which abound in the area of Miocene deposition in southern Trinidad. Their effects can be studied during their present sporadic activity, which is both subaerial and submarine (Kugler, 1938). The materials ejected by these volcanoes comprise a size range from boulders to clay, and they generally indicate Miocene, Oligocene, Eocene, and Cretaceous sediments as provenance. Hence these outbursts supply the correct type of material at the right place and time (continuous throughout the Miocene to Recent at least). The volume of material supplied is not easy to compute, but, in the author's opinion, at least some 10

per cent of Miocene sediments could have come from such a source.

The mud volcano ejectmenta supply two dominant sizes of material to the Miocene sediments: a large amount of clay and some much smaller amount of sand and gravel; the latter has been stressed in this discussion only because it is easier to trace petrographically and is more obvious to both geologists (presence of pebbles and boulders of older rocks) and paleontologists (source of the so-called "reworked" faunas of Oligocene, Eocene, and Cretaceous "age" in clays at the base of Miocene sedimentary cycles). It is probable that modern techniques of analysis for clay-size minerals could confirm the presence of Oligocene clay detritus in reasonably large amounts at certain horizons in the Trinidad Miocene sediments.

This deviation from the trend is also, therefore, of very great geological importance. These sediments are less well sorted because of volcanic incidents which occurred during their sedimentational interval, and they therefore indicate interference with the normal procedure of deposition after fluvial transport of some 1,000 miles.

When all those deviations are removed from the scatter diagrams, it appears probable that a linear trend would remain over the median size range of 2.5–6ϕ (and perhaps to 8ϕ) in the Miocene sediments of southern Trinidad, and for this reason such an approach is considered justifiable by the writer. Variations from this statistically and petrographically valid trend are of some importance, and, when experimental variation is removed, any deviation that remains is capable of explanation on a geological basis similar to that outlined in the above cases of groups 3 and 4. It is now possible to examine the variation in trends in space and time over the area of southern Trinidad throughout the Miocene.

TABLE 3

STRATIGRAPHIC SAMPLING PATTERN IN MIOCENE SEDIMENTS OF WELL L

Member	No. of Samples	Feet Cored	Feet per Sample	Remarks
Upper Morne L'Enfer	16	88	5.5	Incompletely represented
Lower Morne L'Enfer	172	1,628	9.5	Well represented
Forest sands	153	1,357	8.9	Well represented
Cruse (Upper)	58	435	7.5	50 per cent represented
Cruse (Middle)	75	650	8.6	Well represented
Cruse (Lower)	57	815	14.2	Well represented
Lengua				Not represented
Total Miocene	531	4,973	9.3	Approximately two-thirds represented

STRATIGRAPHICAL VARIATION IN TREND

The scatter diagram illustrated in figure 5 includes samples from five distinct members of the Miocene succession, and the exact stratigraphic range of sampling is summarized in table 3. It can be seen that, in terms of stratigraphic members, five are adequately represented, and the sampling interval runs approximately one sample to 10 feet, irrespective of member, i.e., the sampling is statistically random and has no strati-

graphic bias. The linear trends for the five members adequately sampled from well L are displayed in figure 6, and the statistics are tabulated in table 4. From the figure and the table it can be seen that trends *1A*, *1B*, *2*, and *4* are reasonably consistent and that the variations are sufficiently small to be neglected. The Upper Cruse samples, however, show a trend (no. *3* in fig. 6) which, although parallel to the other trends, is moved toward the finer sizes. No sand samples

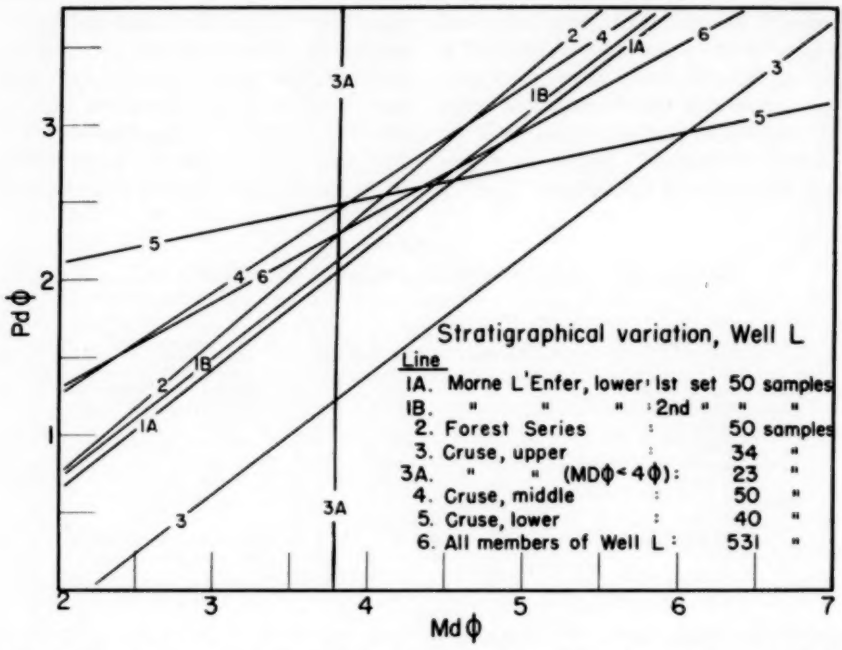

FIG. 6.—Size versus sorting relationship, Miocene, southwestern Trinidad. Stratigraphical variation in well L.

TABLE 4

STRATIGRAPHIC VARIATION IN WELL L

Member	Least-Squares Trend	*Sy*	*r*	No. of Samples	*P*
Morne L'Enfer Lower	$PD_\phi = 0.7832 Md_\phi - 0.9280$	0.75	0.8722	50	<0.001
Morne L'Enfer Lower	$PD_\phi = 0.7790 Md_\phi - 0.8360$	0.80	0.7831	50	<0.001
Forest series	$PD_\phi = 0.8617 Md_\phi - 0.9823$	0.73	0.7176	50	<0.001
Cruse, Upper	$PD_\phi = 0.7681 Md_\phi - 1.6918$	NC*	0.6957	34	<0.01
Cruse, Upper†	$PD_\phi = 127.06 Md_\phi - 479.7864$	NC	NC	23	.....
Cruse, Middle	$PD_\phi = 0.6651 Md_\phi - 0.0751$	0.84	0.7668	50	<0.001
Cruse, Lower	$PD_\phi = 0.209 Md_\phi + 1.6957$	0.98	0.3112	40	<0.05
All horizons	$PD_\phi = 0.5574 Md_\phi + 0.1754$	0.77	0.7585	531	<0.001

* NC, not computed.

† Twenty-three selected samples of Upper Cruse sands with $Md_\phi < 4\phi$.

such as those present in the remainder of the succession occur at this horizon, and so the size effect is marked in this trend. The only sands which are represented in these samples are from the peculiarly uniform body of sand already discussed. Further to illustrate the effect of this sand on the trend, all samples from this interval with Md_ϕ greater than 4ϕ were eliminated, and the remainder (23 samples) were used to calculate a trend line (no. *3A*, fig. 6). It is clear that mathematically there is no association between size (Md_ϕ) and sorting (PD_ϕ) in this sand. Petrographically the sand shows a tremendous range in PD_ϕ for a very small change in Md_ϕ, and this is, of course, emphasized in the peculiar trend line. The only component varying in this case is $P_{90\phi}$, and small changes in the amount of clay-size material would, of course, have a very great effect on the $P_{90\phi}$, which could, in part, be the explanation of the peculiar variation. It was also found, however, that in this sand the value of experimental error far exceeded that of variation from sample to sample in terms of Md_ϕ, and hence, by the method of size analysis used, no variation in Md_ϕ, $P_{10\phi}$, or $P_{90\phi}$ could be established as a valid change in size from sample to sample. This, of course, serves to emphasize the uniformity in this unusual sand body.

Another exceptional trend line is shown by samples from the Lower Cruse member, and here the slope of the trend line (no. *5*, fig. 6) has changed considerably. By analogy and from our earlier discussion, this implies the presence of clay because the slope is less steep than the average trend (no. *6*, fig. 6). It is no surprise, therefore, that in well L most of the Lower Cruse samples comprise silt and clay. The larger part of the reduction in slope in this case is also illusory because the clays had in many cases flocculated, falling well below the sorting measure predicted for their size. Hence this trend line is almost entirely due to experimental error in terms of lack of control in the experimental technique.

The trend line for the complete sequence is given in line *6*, and the combined effects of trend lines *3* and *5* have resulted in only a slight change in slope in trend *6* as compared with lines *1A*, *1B*, *2*, and *4*. As would be expected, there is very little displacement due to size. On this basis it is probable that trend lines *1A* and *1B* are nearer the "true" relationship between size and sorting than are any other trends computed. This is borne out by the fact that less flocculation occurred in the clayey samples from this horizon.[3]

It is possible to carry this analysis of stratigraphic variation in trend a step further, as a number of other sets of samples have been taken from similar horizons in other wells from the same field. In this way it is also possible to illustrate the variation within members over a very restricted geographical area.

The trend lines of size versus sorting in the Forest series from two wells and the Middle Cruse from three wells are illustrated in figure 7, and the relevant statistics are summarized in table 5. Two other trends are inserted for comparison, the first representing the average trend for all horizons in well L and the second trend based on the average of two sets of samples from the Lower Morne L'Enfer horizon from well L, the "true" trend line. It can be seen from the scatter diagram that, in terms of size (i.e. position

[3] Owing, no doubt, to the base saturation of the clays and their associated fluids, as these samples for the larger part lie in the ground-water horizon rather than in the highly saline oil-bearing horizons.

on the graph), there is no significant variation between trends, but they can be grouped into two sets on the basis of slope. The steeper set—lines *1*, *2*, and *7*—are sandier sediments and approximate the "true" trend or relationship, whereas the others—lines *3*, *4*, *5*, and *6*—are all less steep. It would, of course, be tempting to infer that the trends for the Forest series are different from those in the Middle Cruse of these wells; but, by inserting lines *6* and *7*, it can be seen that the difference is probably much less significant, being probably attributable to

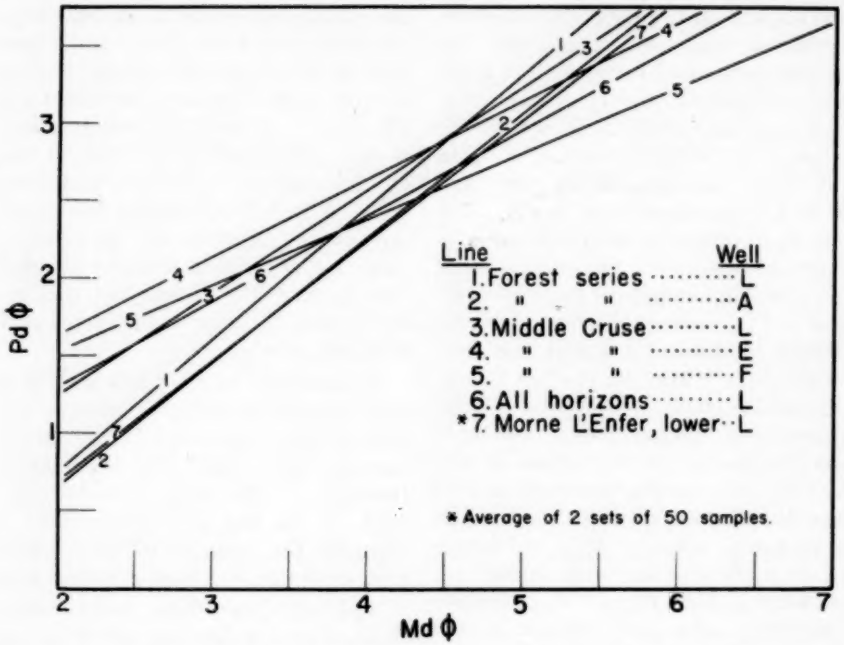
Pd ϕ
Md ϕ
Line
Well
1. Forest series L
2. " " ..... A
3. Middle Cruse L
4. " " ..... E
5. " " ..... F
6. All horizons L
*7. Morne L'Enfer, lower .. L
* Average of 2 sets of 50 samples.

FIG. 7.—Size versus sorting relationship, Miocene, southwestern Trinidad, variation within horizons

TABLE 5

STRATIGRAPHIC AND LIMITED GEOGRAPHIC VARIATION WITHIN MEMBERS

Well	Horizon	Least-Squares Trend	*Sy*	*r*	No. of Samples	*P*
A.....	Forest series	$PD_\phi = 0.806\ Md_\phi - 0.954$	0.49	0.9085	20	<0.001
L.....	Forest series	$PD_\phi = 0.8617 Md_\phi - 0.9823$	0.73	0.7176	50	<0.001
E.....	Middle Cruse	$PD_\phi = 0.5019 Md_\phi - 0.6296$	0.70	0.7423	50	<0.001
F.....	Middle Cruse	$PD_\phi = 0.4283 Md_\phi - 0.6776$	0.51	0.9207	50	<0.001
L.....	Middle Cruse	$PD_\phi = 0.6651 Md_\phi - 0.0751$	0.84	0.7668	50	<0.001
L.....	All horizons	$PD_\phi = 0.5574 Md_\phi + 0.1754$	0.77	0.7585	531	<0.001
L.....	Morne L'Enfer Lower*	$PD_\phi = 0.7811 Md_\phi - 0.8820$	0.78	0.8276	100	<0.001

* Average of two sets of 50 samples.

more clays and more flocculation of the clays from the Middle Cruse than those from the Forest series.

In summary, then, any variation from horizon to horizon stratigraphically or within horizons and over geographic limits of 1–2 miles is not brought out in these examples, and, as far as the present analysis is concerned, all stratigraphic variations are to be attributed to other than purely geological factors.

two are based on surface samples and the third is from a well. The geographic range from east to west is about 50 miles.

The trend lines computed from these sets of data are exhibited in figure 8, and the statistical parameters are summarized in table 6. Trends *2*, *3*, and *5* in figure 8 are parallel, but, from the positions of the trends, *3* and *5* are from samples which are finer grained than *2*. In terms of geographic position the sur-

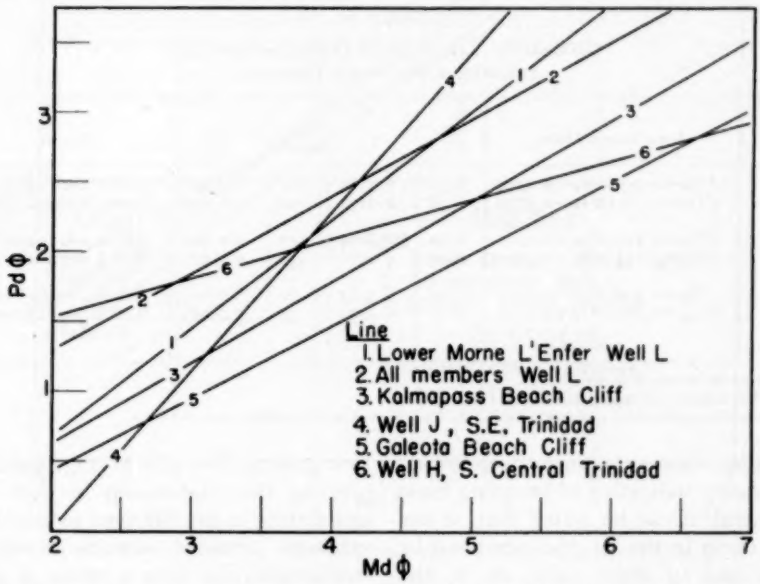

FIG. 8.—Size versus sorting relationship, Miocene, southern Trinidad, geographical variation

GEOGRAPHIC VARIATION IN TREND

The Miocene sediments of southern Trinidad extend from the extreme west to the eastern land limits of the island. The examples so far discussed are nearly all from a single field (Forest Reserve, in southwestern Trinidad). One example has also been computed on the basis of subsurface samples from south-central Trinidad and three from the extreme southeastern part of the island. Of these, face samples from southeastern Trinidad are finer grained than those from well L in southwestern Trinidad, but the relationship between size and sorting is otherwise almost identical. Trend *1*, which represents the selected "true" trend from studies of horizons in well L, is steeper than all three of these trends, and, as it nearly coincides in position with trend *2*, it also contains coarser-grained sediments than *3* or *5*.

This change in grain size from east to west is confirmed by comparing sets of samples from the two areas in terms of their cumulative curves.

Trend *4* in figure 8 represents the relationship between size and sorting (PD_ϕ) based on 43 subsurface samples from well J in southeastern Trinidad. It is somewhat steeper than the other trends in the figure, but its general position coincides with trends *1* and *2*. On the basis of earlier comments, such steep trends are usually indicative of sampling bias; in general, it can be stated that, if few clays occur in the set, the trend will be steep, and, if many clays occur, the trend will be much less steeply inclined. On the basis of individual grain-size distribution, it can be stated that the samples from well J are all very fine sands or coarse silts.

Trend *6* is much less steeply inclined and indicates quite clearly the presence of a large number of clay samples in the set, and, by analogy with the discussion on well L, these clays have flocculated during analysis. Examination of the individual cumulative curves confirms this conclusion. It is noteworthy that in such cases the correlation coefficient is low and not statistically significant.

On the basis of these examples it can be seen that, while the majority of samples from the southeast of Trinidad are finer grained than those in the southwest of Trinidad, the relationship between median size and percentile sorting remains consistent. Any variations from the general trends are due to either sampling or experimental error or both.

TABLE 6

GEOGRAPHIC VARIATION IN TREND IN MIOCENE SEDIMENTS, SOUTHERN TRINIDAD

Well	Least-Squares Trend	*Sy*	*r*	No. of Samples	*P*	Remarks
L	$PD_\phi = 0.5574 Md_\phi + 0.1754$	0.77	0.7585	531	<0.001	All horizons well L
L	$PD_\phi = 0.7811 Md_\phi - 0.8820$	0.78	0.8276	100	<0.001	Lower Morne L'Enfer well L*
K†	$PD_\phi = 0.5797 Md_\phi - 0.5311$	0.51	0.6264	37	<0.001	Kalmapass Beach Cliff
J	$PD_\phi = 1.144\ Md_\phi - 2.2752$	0.48	0.6566	43	<0.001	Well J, southeastern Trinidad
M†	$PD_\phi = 0.5194 Md_\phi - 0.6056$	0.49	0.5173	67	<0.001	Galeota Beach Cliff
H	$PD_\phi = 0.2812 Md_\phi + 0.9688$	0.48	0.3494	31	>0.05‡	Well H, south-central Trinidad

* Average of two sets of 50 samples.
† Surface samples from southeastern Trinidad.
‡ Correlation coefficient not statistically significant; *P* nearly equal to, but slightly greater than, 0.05.

In summary, then, the average trend expressing the relationship between size and sorting in the Miocene sediments of southern Trinidad remains consistent stratigraphically over a range of some 7,000 feet and geographically over a distance of some 50 miles.

THE OLIGOCENE DEPOSITS OF SOUTHERN TRINIDAD

The Oligocene deposits of southern Trinidad have been subdivided into two facies, the Nariva "formation," a nearshore sediment, and the Cipero "formation," representing a shallow-water marine environment (Stainforth, 1948). The subsequent analysis refers solely to the Herrera sands and associated clays

from the Cipero facies of the Oligocene. The Herrera sands are quartzose graywackes with much less matrix than the Miocene sands. In addition, they are coarser grained and contain a relatively large proportion of gravel (conglomerates). The most striking contrast between Miocene and Oligocene deposits is, however, the dominance of silt in the Miocene and the paucity of silt in the Oligocene. These two groups of sediments are clearly representative of two very different environments of deposition.

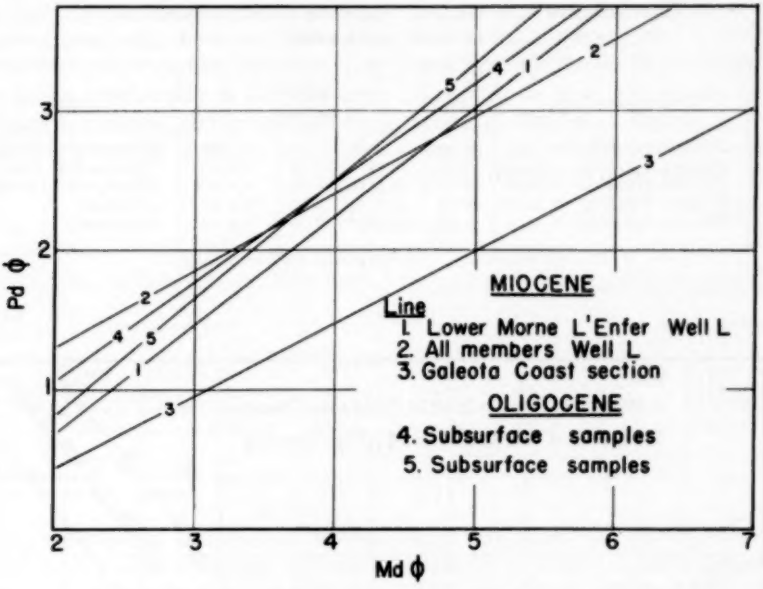

FIG. 9.—Size versus sorting relationship, Miocene and Oligocene, southern Trinidad

For the purpose of comparison, the first set of Oligocene samples has been chosen from sequences which do not contain gravel grain sizes in any quantity. Despite the selection, the Oligocene sediments are still, on the average, coarser than their lithologic equivalents in the Miocene series. The least-squares trends of a number of Miocene and Oligocene samples are illustrated in figure 9, and the relevant statistics are summarized in table 7. Superficial examination of the Oligocene trends shows their similarity to the average trend from well L in the Miocene and their even closer identity with the sandier sediments of the "true" trend. The two Oligocene trend lines (lines *4* and *5*, fig. 8) are displaced toward coarser average sizes, and their steepness reflects the preponderance of sands in the sets of samples chosen. Here again, then, over the range of size from approximately 2.5ϕ (0.177 mm.) to 6ϕ (0.0156 mm.) median diameters the percentile sorting becomes poorer with decreasing grain size.

Taking into account the sets of samples with gravel fractions shows an

entirely new type of distribution (fig. 10). In this scatter diagram it is clear that no single linear trend can adequately represent the relationship. Continuing the assumption of linearity as a first approximation, two trends are necessary, and they have been computed and inserted in figure 10; the relevant statistics for the four sets of Oligocene data are given in table 8.

From these data it can be seen that over a range in median diameter of 2.5–6ϕ the percentile sorting becomes poorer (PD_ϕ increases in absolute value) with

TABLE 7

COMPARISON OF MIOCENE AND OLIGOCENE RELATIONSHIPS OVER THE SAME SIZE RANGE

Trend*	Least-Squares Trend	Sy	r	No. of Samples	P	Horizon
1	$PD_\phi = 0.5574 Md_\phi + 0.1754$	0.77	0.7585	531	<0.001	Miocene SW. Trinidad
2	$PD_\phi = 0.7811 Md_\phi - 0.8820$	0.78	0.8276	100	<0.001	Miocene SW. Trinidad
3†	$PD_\phi = 0.5194 Md_\phi - 0.6056$	0.49	0.5173	67	<0.001	Miocene SE. Trinidad
4	$PD_\phi = 0.7083 Md_\phi - 0.3420$	0.55	0.5913	30	>0.01	Oligocene
5	$PD_\phi = 0.8500 Md_\phi - 0.9063$	0.49	0.7092	30	<0.001	Oligocene

* See fig. 9.

† Surface samples, Galeota coast section = M of table 6.

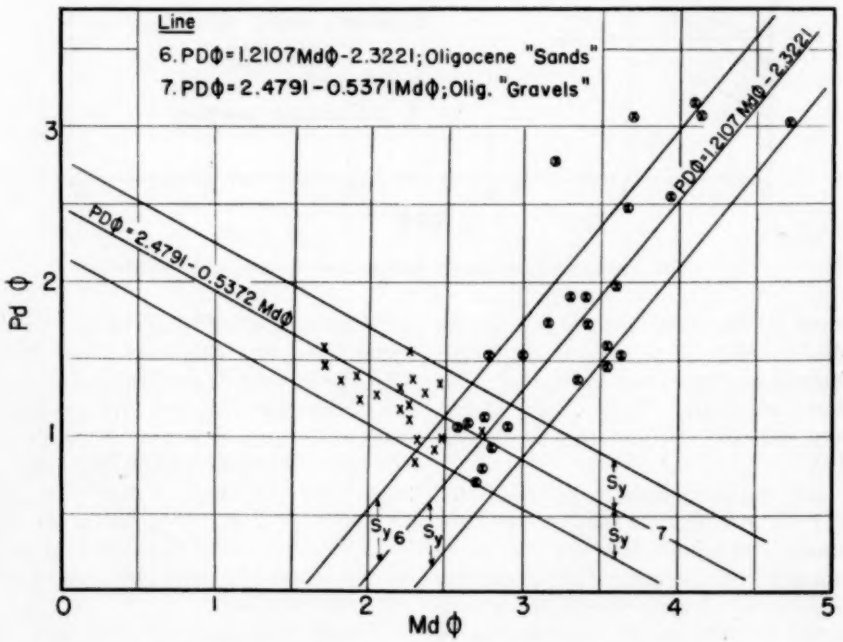

FIG. 10.—Size versus sorting relationship, Oligocene, southern Trinidad

decrease in grain size (increase in value of phi median); from median diameter 2.5–1.0ϕ, however, this relationship is reversed, and it can be seen that, with increasing median diameter (decreasing phi value), the percentile sorting becomes poorer. It seems apparent, therefore, that sediments of median diameter around 2.5ϕ (0.177 mm.) are the best sorted, whereas the sorting becomes poorer with both increasing and decreasing size from this diameter. This conclusion refers solely to the Oligocene sediments analyzed and is confined to a grain-size range of 1–6ϕ (0.500–0.0156 mm.).

ment which is coarse grained and contains gravel grain sizes in considerable proportion. Superficial examination of a few thin sections of this material suggests that it is also a graywacke. This is another marine sediment, but one which suggests a very different environment from either Miocene or Oligocene of Trinidad. Through the courtesy of the late Dr. A. Senn, the author was enabled to collect a representative series of samples from the Lowes Ridge type section (Senn, 1940, p. 1557) of the Scotland series in Barbados during a vacation visit in 1942. A plot of these samples in a scatter diagram is illustrated in figure 11,

TABLE 8

TRENDS IN THE OLIGOCENE OF SOUTHERN TRINIDAD

Trend Line*	Least-Squares Trend	*Sy*	*r*	No. of Samples	*P*	Remarks
4.....	$PD_\phi = 0.7083 Md_\phi - 0.3420$	0.55	0.5913	30	<0.01	Oligocene sands
5.....	$PD_\phi = 0.8500 Md_\phi - 0.9063$	0.49	0.7092	30	<0.001	Oligocene sands
6.....	$PD_\phi = 1.2107 Md_\phi - 2.3221$	0.44	0.8115	28	<0.001	Oligocene sands
7.....	$PD_\phi = 2.4791 - 0.5372 Md_\phi$	0.31	−0.3788	20	<0.10†	Oligocene gravels

* See fig. 10.
† Not statistically significant.

The reversal in association between phi median diameter and phi percentile diameter is shown by the change in sign of the correlation coefficient. It must be emphasized, however, that the level of statistical significance ($P < 0.10$) attached to the negative correlation coefficient (−0.3788, trend line 7) indicates that such a relationship would arise by chance from random sampling of an unrelated population about 10 times in 100. Statistically, therefore, this set of samples is insufficient to establish the reversal of trend.

THE EOCENE SEDIMENTS OF BARBADOS

The Scotland series of Eocene age in Barbados (Senn, 1940) is another sedi-

and here again the distribution cannot logically be fitted to a single linear trend. The two trends, together with the two Oligocene trends, are inserted in the figure. Here again the sediments around 2.5–2.75ϕ (0.177–0.149 mm.) appear to be the best sorted, and sorting becomes poorer when it proceeds toward coarser and finer sizes. The relevant statistics are summarized in table 9, and it can be seen that the trend is reversed and that, while still not statistically significant at the level chosen for this report ($P < 0.02$), such a value of r would arise by chance from an uncorrelated population less than twice in 100 examples. The similarity of the Trinidad Oligocene trend to that of the Barbados Eocene

serves further to confirm the reality of the reversal of trend. Increase in the number of coarser-grained sediments would almost certainly bring this level of significance into line with the others tabulated in this report.[4]

THE SUPERFICIAL GRAVELS OF THE GUIANAS

The Superficial Gravels of both Dutch and British Guiana have been penetrated in two deep wells, and cores from both wells have been analyzed for grain size. The sediments from the British Guiana well have been described in detail (Griffiths, 1942); on the basis of mineral composition they are arkoses, but their depositional environment most nearly resembles that of the Barbados Eocene sediments. The Guiana gravels are considered to have been "dumped" after a short period of transport; the grain size varies from gravels to clays, and the size-sorting relationships on the basis of two sets of samples, one from each well, are displayed in figure 12. The relevant statistics are given in table 10, together with representative data from the Oligocene and Miocene of Trinidad and the Eocene of Barbados. The size versus sorting relationship in the British Guiana gravels is somewhat similar to that in the Miocene in slope but is displaced markedly toward coarser average grain sizes, as would be expected. The general relationship suggests that percentile sorting becomes poorer with decrease in grain

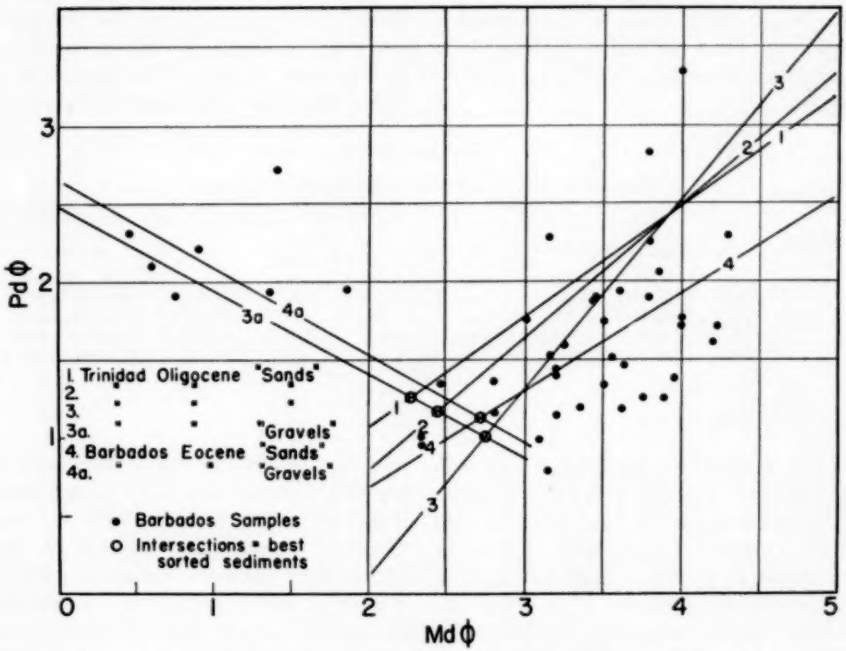

FIG. 11.—Size versus sorting relationship, Oligocene, Trinidad, and Eocene, Barbados

[4] This problem is being actively pursued at present, and initial results offer confirmation of these conclusions.

size over the whole range tested (0–5ϕ, 1.0–0.0312 mm.). The average grain size of the Guiana gravels is very much coarser than that of the Miocene and Oligocene sediments of Trinidad but is reasonably similar to the average size of the Barbados samples. No reversal of trend around a median diameter of 2.5ϕ (0.177 mm.) is noted in the Guiana gravels. This suggests that the deposition of the Guiana gravels was different from that of the Barbados Eocene deposits, despite their similarity in average grain size.

The most outstanding difference between the Guiana samples and those of Trinidad and Barbados is in the value of the standard error of estimate (*Sy*). The Guiana gravels have much larger *Sy* values than any of the other trends, and this appears to be true of both Dutch and British Guiana samples. In a geological sense it appears reasonable to infer that these departures are also significant. Considering only the trends over the sand fractions from the Miocene of southern Trinidad reduces the value of *Sy* (*Sy* generally less than 0.50), and

TABLE 9

COMPARISON OF BARBADOS EOCENE AND TRINIDAD OLIGOCENE RELATIONSHIPS

Trend Line*	Least-Squares Trend	*Sy*	*r*	No. of Samples	*P*	Remarks
1. . . .	$PD_\phi = 0.7083 Md_\phi - 0.3420$	0.55	0.5913	30	<0.01	Oligocene "sands"
2. . . .	$PD_\phi = 0.8500 Md_\phi - 0.9063$	0.49	0.7092	30	<0.001	Oligocene "sands"
3. . . .	$PD_\phi = 1.2107 Md_\phi - 2.3221$	0.44	0.8115	28	<0.001	Oligocene "sands"
3A. . .	$PD_\phi = 2.4791 - 0.5372 Md_\phi$	0.31	−0.3788	20	<0.10	Oligocene "gravels"
4. . . .	$PD_\phi = 0.6193 Md_\phi - 0.5596$	0.44	0.5465	34	<0.01	Barbados "sands"
4A. . .	$PD_\phi = 2.6474 - 0.5608 Md_\phi$	NC†	−0.7620	10	<0.02	Barbados "gravels"

* See fig. 11.

† NC, not computed.

The association of average size (Md_ϕ) and percentile sorting (PD_ϕ) in terms of the correlation coefficient is of the same order in the British Guiana samples as that in the Miocene and Oligocene of Trinidad and the Eocene of Barbados over the size range of 2.5–5ϕ. It is statistically significant at the 1 per cent level ($P < 0.01$). In the Dutch Guiana samples the level of significance is lower ($P < 0.05$), but this is probably due to the fact that fewer samples are available (16 as compared with 28, respectively). The association is therefore statistically significant and, in addition, by analogy with the Trinidad and Barbados samples, geologically significant.

the paucity of clays in the sets of Oligocene samples also exemplifies this feature ($Sy = \pm 0.5$). The same is true of the Barbados gravelly deposits ($Sy = 0.50$). In other words, sandy deposits show a size-sorting relationship, with departures from the trend measured by *Sy* of less than 0.5. The median diameters of the Guiana gravels vary between 0 and 4ϕ, and yet even in these sandy sediments the *Sy* is approximately twice as large ($Sy = \pm 1.0\phi$), indicating that the departures from the trend are twice as large. The significance of this contrast cannot be categorically stated, and its interpretation will therefore be treated in the discussion of the results (p. 240).

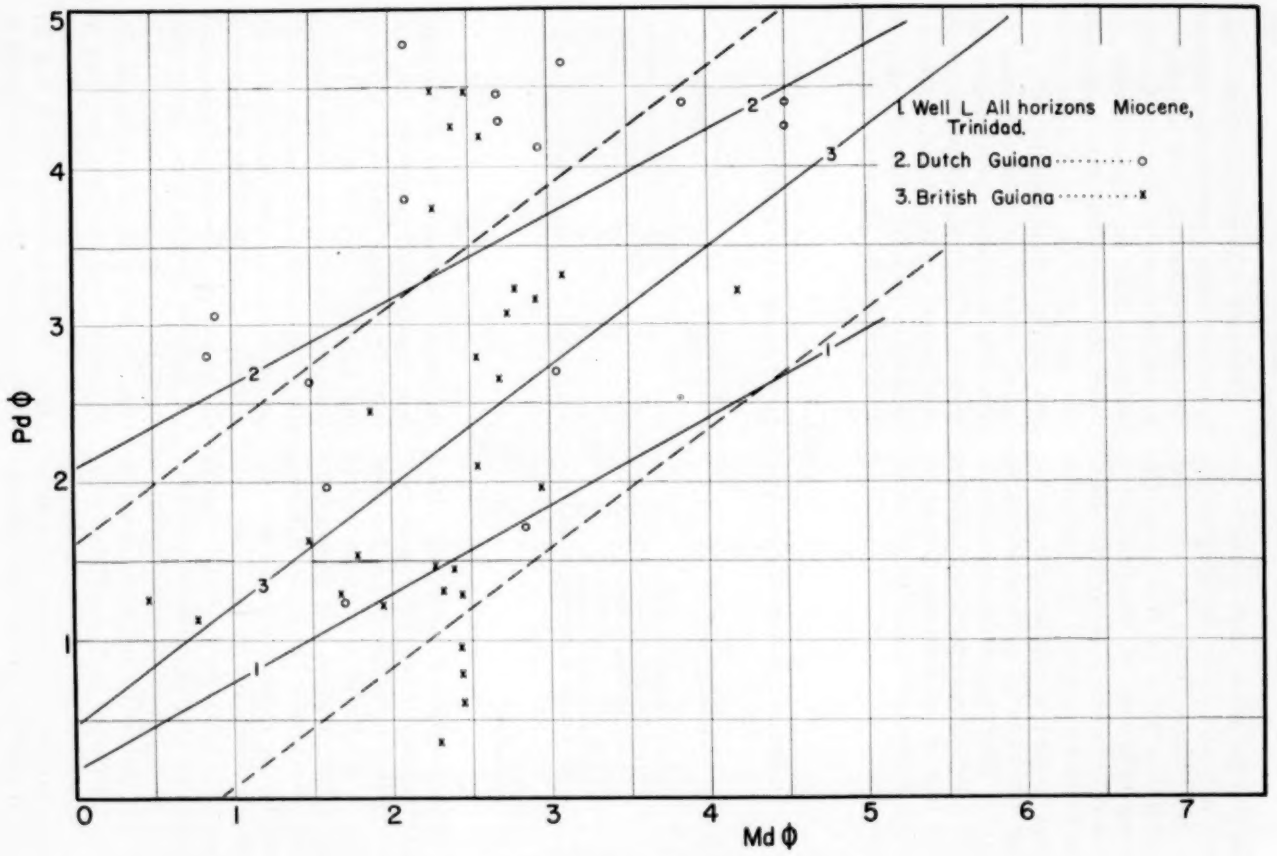

FIG. 12.—Size versus sorting relationship, Guiana gravels

Discussion of the Results

Interpretation of the Trend

The relationships between size and sorting in the sediments herein described lead to the following general conclusions:

1. Over the range in average size (Md_ϕ) of 0–6ϕ (1.0–0.0156 mm.) the percentile sorting (PD_ϕ) varies with size.

2. Between size limits of 0 and 2.5ϕ (1.0–0.177 mm.) sorting becomes poorer (PD_ϕ increases in value) as the average size becomes larger (Md_ϕ decreases in value).

3. Between 2.5 and 6ϕ (0.177–0.0156 mm.) sorting becomes poorer (PD_ϕ increases in value) as the average size becomes smaller (Md_ϕ increases in value).

4. The best-sorted sediments lie around 2.5ϕ (0.177 mm.) in average diameter and possess percentile sorting values generally less than 1ϕ (i.e, $PD_\phi < 1\phi$).

Such conclusions are confirmed by Trask's observations on size and sorting based on empirical studies of sediments (1932) and on Inman's analytical approach based on the hydrodynamics of water transport (1949). It must be remembered that all these remarks concern sediments which are water-borne and water-deposited. From preliminary testing, similar relationships exist in wind-borne and wind-deposited sediments, but glacial and gravitational deposits show very different relationships (usually vague to none) between size and sorting.

It would appear, therefore, that a generalized relationship between size and sorting exists in water-borne and water-deposited material, and even on the basis of this preliminary testing the conclusions appear to be amply justified. It also seems reasonable to assume a linear trend over restricted size intervals, but only so long as the limitations are fully recognized and taken into account in any conclusions which may be drawn from the data so tested. By using millimeter and semilogarithmic plots, Inman (1949, fig.1) obtained a curvilinear trend for various kinds of deposits; and, if the trends displayed in the present work are replotted on a semilogarithmic scale in terms of millimeter units, similar curvilinear relationships are, of course, obtained (fig. 13). The use of the phi scale, therefore, tends to simplify the analysis and does not distort the conclusions.

TABLE 10

Comparison of the Guiana Gravels with Other Caribbean Sediments

Trend Line*	Least-Squares Trend	Sy	r	No. of Samples	P	Remarks
1	$PD_\phi = 0.7523 Md_\phi + 0.4681$	1.14	0.5555	28	<0.01	British Guiana
2	$PD_\phi = 0.5360 Md_\phi + 2.0788$	0.95	0.5238	16	<0.05	Dutch Guiana
3	$PD_\phi = 0.5574 Md_\phi + 0.1754$	0.77	0.7585	531	<0.001	Miocene, Trinidad
4	$PD_\phi = 0.7811 Md_\phi - 0.8820$	0.78	0.8276	100	<0.001	Miocene, Trinidad
5	$PD_\phi = 0.7083 Md_\phi - 0.3420$	0.55	0.5913	30	<0.01	Oligocene, Trinidad
6	$PD_\phi = 1.2107 Md_\phi - 2.3221$	0.44	0.8115	28	<0.001	Oligocene, Trinidad
7	$PD_\phi = 2.4791 - 0.5372 Md_\phi$	0.31	−0.3788	20	<0.10	Oligocene, Trinidad
8	$PD_\phi = 2.6474 - 0.5608 Md_\phi$	NC†	−0.7620	10	<0.02	Eocene, Barbados
9	$PD_\phi = 0.6193 Md_\phi - 0.5596$	0.44	0.5465	34	<0.01	Eocene, Barbados

* See fig. 12.

† NC, not computed.

TRANSFORMATION OF THE LINEAR TREND FROM PHI TO ITS EQUIVALENT IN MILLIMETERS[5]

Krumbein derived the phi scale from the relationship $\xi = 2^{-\phi}$, where ξ = diameter in millimeter units (1938). Therefore,

$$-\phi = \frac{\log_{10} \xi}{\log_{10} 2};$$

and, as

$$PD_\phi = \frac{P_{90\phi} - P_{10\phi}}{2},$$

this may be similarly transformed to

$$PD_\phi = \frac{(-\log_{10} P_{90}) / (\log_{10} 2) + (\log_{10} P_{10}) / (\log_{10} 2)}{2},$$

which may be reduced to

$$\frac{-\log_{10} (P_{90}/P_{10})}{2 \log_{10} 2} = \frac{\log_{10} PDg}{\log_{10} 2},$$

where PDg is the geometric percentile deviation in millimeter units and $PDg = \sqrt{P_{10}/P_{90}}$ (see analogous transformation of QD_ϕ to $So = QDg$; Krumbein and Pettijohn, 1938, p. 234). Therefore,

$$PD_\phi = \frac{\log_{10} PDg}{\log_{10} 2} \text{ or } 0.301 PD_\phi = \log_{10} PDg.$$

But the geometric percentile deviation $PDg = \sqrt{P_{10}/P_{90}}$ (millimeter units), and it follows that $\log \sqrt{P_{10}/P_{90}} = 0.301\ PD_\phi$ or $\sqrt{P_{10}/P_{90}} = 10^{0.301 PD_\phi} = 2^{PD_\phi}$. Hence $PDg = 2^{PD_\phi}$; substituting this relationship in the linear trend $PD_\phi = bMd_\phi - a$, we get

$$\sqrt{\frac{P_{10}}{P_{90}}} = 2^{bMd_\phi - a} = \frac{2^{bMd_\phi}}{2^a};$$

but

$$Md_\phi = \frac{-\log_{10} Md_\xi}{\log_{10} 2} = -\log_2 10 \cdot \log Md_\xi;$$

it therefore follows that

$$PDg = \sqrt{\frac{P_{10}}{P_{90}}} = \frac{2^{-b \log_2 10 \cdot \log Md_\xi}}{2^a}$$

$$= \frac{10^{-b \log Md_\xi}}{2^a} = \frac{Md_\xi^{-b}}{2^a} = \frac{1}{2^a Md_\xi^b}.$$

Therefore, in millimeter units, $PDg = 1/2^a Md_\xi^b$. Commencing with $PD_\phi = bMd_\phi + a$ results in the millimeter equivalent, $PDg = 2^a/Md_\xi^b$.

The generalized equations in terms of millimeter units are exhibited as actual trends in figure 13, using the empirically derived values for two sets of samples from the Miocene sediments of southern Trinidad, two from the Oligocene, and two from the Eocene of Barbados. It will be readily appreciated that the use of phi units and the assumption of linear trends over restricted size ranges serve to simplify materially the resulting analytical treatment and its subsequent interpretation.

SOME IMPLICATIONS OF THE TREND IN SIZE VERSUS SORTING

Commencing, now, with the linear trend $PD_\phi = bMd_\phi - a$, we can transform this trend to $PD_\phi = bMd_\phi$ by changing the origin to some value of PD_ϕ equal to $-a$. This equation can then be further reduced to

$$\frac{P_{90\phi} - P_{10\phi}}{2} = bMd_\phi \text{ or } P_{90\phi} - P_{10\phi} = 2\,bMd_\phi;$$

but, as Md_ϕ is a close approximation to the modal diameter in the sediments which gave rise to this relationship (e.g., the Miocene sediments of southern Trinidad), then $P_{90\phi} - P_{10\phi} = kMo_\phi$, where k

[5] I am indebted to Professor E. Johnson for this mathematical operation.

is some constant approximately equal to $2b$. In simple terms, $P_{90\phi} - P_{10\phi} \propto f(Mo_\phi)$, or the spread between the phi percentiles varies as a function of the phi modal diameter. For purposes of exploration let us assume that the current velocity remains constant and that, therefore, Mo_ϕ is reasonably constant. Then any variation in $P_{90\phi}$ must be accompanied by a similar variation in velocity and bearing a sand-size material as its preferred load (Mo_ϕ around very fine sand size in, say, the Mississippi River) there is a sudden influx of clay-sized material into the stream, then some of the coarser sand sizes must be deposited. This is, of course, a much simplified and idealized case but offers, in part at least, a possible explanation for local deposition of fine and medium

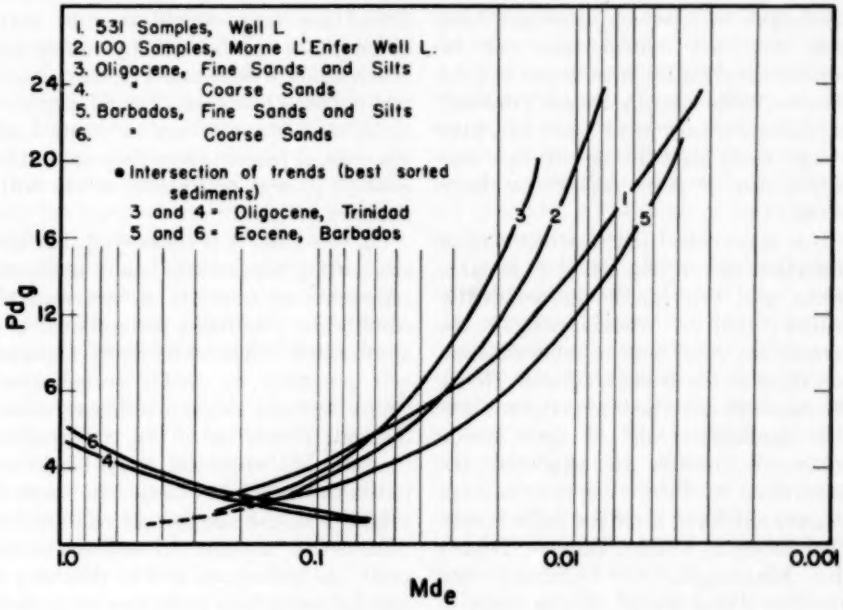

FIG. 13.—Size versus sorting relationship, Md_ϕ versus PD_ϕ in millimeter units

$P_{10\phi}$. For example, suppose $P_{90\phi}$ is increased from 2 to 4ϕ; then $P_{10\phi}$ must also be increased 2ϕ units to maintain the relationship. In terms of sedimentation, increasing $P_{90\phi}$ means decreasing the diameter of P_{90} in millimeters, i.e., adding clay size to the material. Similarly, increasing $P_{10\phi}$ means decreasing the diameter value of P_{10} in millimeters. Such an implication suggests that if in a stream traveling with uniform average sand in streams by collapse of clay-sized stream banks.

The converse of this operation is not so easily visualized, i.e., suppose similar initial conditions and an influx of coarse sand. Then by similar reasoning the P_{10} in millimeters increases and the $P_{10\phi}$ decreases in value. For uniform conditions to hold, $P_{90\phi}$ must also decrease in value, and some clay therefore must drop out. It seems more reason-

able to suppose that the stream would fail to pick up the coarser sand and, therefore, that such additions would have little ultimate effect on the size distribution of the transported material. In actual practice it seems even more probable that sudden local additions of material would affect the average velocity and, therefore, the value of Mo_{ϕ}; but, as the size of the materials ultimately deposited by such large streams as the Mississippi is relatively constant (and such restricted distributions can be demonstrated for the Mississippi and the Miocene sediments of southern Trinidad, for example), then local conditions play a very small part in the ultimate size distribution of delta deposits of large rivers.

It is also evident that one of the most important loci of deposition is in large deltas, and this can be independently demonstrated by examination of the present-day conditions of sedimentation and those of the geologic column. Hence this depositional type is of very considerable significance, and, if approximate volumetric controls are estimated, the rivers must be of the order of 1,000 miles long for a delta of some 200 miles across. The Amazon, Congo, Ganges, Hwang Ho, Mississippi, Nile, Orinoco, and Yangtze Kiang are all of this order of magnitude, whereas the Indus, Irrawaddy, and Rhine are only a little smaller—both in length and size of delta. In all these cases it appears that the transported material has had a very long time interval during which relatively undisturbed conditions have operated on the materials to produce sediments of similar size distributions. The size-sorting relationships in only two have been examined, but it is obvious that under such physical conditions as those represented by these deposits such a relationship is likely to reach its maximum development. Indeed, it appears probable that delta deposits represent as near equilibrium conditions as we can ever hope to expect within the average sedimentational environment. The size-sorting relationship in these cases is therefore as well expressed as we can expect. It may be objected that a more efficient agent, acting for a shorter time interval, would bring about a similar equilibrium, but the actual size range would then be very different, as can be seen by examining beach sands or dune sands (Inman, 1949; and author's observation), and, furthermore, such deposits are far more conspicuous at present than they are in the geologic past as represented in the sedimentary column.

On this basis it is considered that the size-sorting relationship in any sediment represents an estimate of how near to equilibrium conditions the sediment has approached. Thus water-borne deposits will generally be better sorted than glacial deposits, owing to different hydrodynamic properties of the transporting medium, but within the water-borne deposits themselves, when the size range is established, the size-sorting relationship reflects the "amount" of sorting the deposit has undergone, and in this way it may become a very useful tool in predicting the environment of deposition. This aspect of size-sorting relationships as a whole is in process of evaluation at present.

In summary, then, it appears likely that the existence of size-sorting relationships in sediments can be used to determine the nature of the transporting medium, and, once this is decided, the perfection of the relationship can be used to determine the kind of depositional environment to some extent. On this basis it is possible to explain the dis-

crepancy in size-sorting relationship between the Trinidad Miocene sediments and the Superficial Gravels of the Guianas. Both are deposited in water, but the Trinidad deposits indicate a much greater perfection in size-sorting relationship on the basis of deviations from the trend (value of the standard error of estimate, *Sy*) than do the Guiana gravels. The gravels have not, therefore, undergone transport, either in intensity or in time, equivalent to the Miocene sediments of Trinidad. The heterogeneity of sizes in the Guiana gravels implies lack of sorting intensity, and hence the transport was short, and reworking after deposition was relatively nil. In other words, the material was "dumped" into the basin of deposition and covered rapidly by succeeding debris. Such was the depositional history deduced from the petrography of these deposits (Griffiths, 1942), and apparently the size-sorting relationship amply confirms those conclusions.

Interpretation could be continued further, even on the basis of the present evidence, but it is prudent to await a thorough evaluation of the size-sorting relationship in terms of transporting fluid and depositional environment by empirically analyzing a much larger number of cases before extrapolating beyond the limits of the present investigation.

Summary and Conclusions

A study of the average grain size in terms of the phi median diameter (Md_ϕ) and uniformity of size distribution in terms of the phi percentile deviation (PD_ϕ) has shown that there exists a definite relationship between size and sorting in many different sediments which are water-borne and water-deposited. By confining attention to restricted grain-size ranges, their relationship may be expressed in terms of linear equations of the general form $PD_\phi = Md_\phi - a$.

Using this relationship in sets of sediments which are known to be water-borne, the deviation from the trend beyond that expected due to "chance" causes (± 1 unit standard error, or $\pm 1\ Sy$ unit) can be used to characterize sediments which have not followed the average relationship. In the sand sizes these deviations have been attributed to incidents in the sedimentational "tectonics" which accompany the deposition of the material and which are presumed to have caused the marked departures from the trends. In the finer sizes such deviations are generally a reflection of flocculation in the clay fractions.

The trends are shown to vary with both sandiness (number of sand and clay samples in the set) and with coarseness (average size of any set of samples). In sets which comprise predominantly sand samples, the trends tend to be steeper than in those which contain many clays, i.e., the slope of the trend line is changed. Differences in average sizes of entire sets of samples lead to translation of the trend without necessarily affecting its slope.

Trends in the Miocene sediments of southern Trinidad show no great variation either stratigraphically over some 7,000 feet or geographically over some 50 miles, apart from variations due to sampling bias, as explained above. Oligocene trends are very similar to the Miocene within the same size ranges, but over the entire range of sizes displayed in the Trinidad Oligocene the size-sorting relationship is curvilinear and indicates that sorting becomes progressively poorer with increasing and decreasing average size from the best-sorted sedi-

ments, which possess diameters around 2.5ϕ (0.177 mm.). Barbados sediments of Eocene age show relationships smiilar to those of the Oligocene of Trinidad.

The Guiana gravels (Pliocene, Griffiths, 1942) show an unusual trend as compared with either the Trinidad or the Barbados samples. Here the trend is less well defined and is not in accord with the size-sorting relationships of either Trinidad or Barbados sediments, although the average size of the sediments is similar to that of the Barbados Eocene samples. The size-sorting trend line is parallel to those of the Trinidad Miocene trends, despite the fact that the size range is more in accord with the Barbados samples.

The *Sy* value in the Guiana samples is twice as large as that in the Trinidad or Barbados samples. This is considered to be due to lack of sorting in the Guiana gravels as compared with those from Trinidad and Barbados. Such a conclusion is in accord with its deduced depositional environment based on other petrographic evidence.

A very brief and hypothetical examination of the implications of the size-sorting relationship suggests that it may prove to be a useful quantitative tool in deciding the nature of the transporting medium and the depositional environment if future investigations now in hand serve to confirm the present findings.

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CAUSES OF THE GLACIAL LAKE STAGES IN SAGINAW BASIN, MICHIGAN[1]

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ABSTRACT

Because the volume of Glacial Grand River varied by several ten thousands of second-feet between the maximum discharge during a retreat of the Saginaw, Huron, and Erie lobes and the minimum discharge during a readvance, the erosional and transportational ability of the river varied. An advance of the ice front, accompanied by slackened erosion in the Grand River outlet for ponded water in front of these lobes, was marked by essentially static lake levels, which are recorded by shore lines. A retreat was marked by erosional deepening of the outlet and falling lake levels. Seven successively lower shore lines were made in the Saginaw Basin while six moraines were being laid down. The seventh moraine, to correspond with the lowest shore line, is presumed to lie submerged in Lake Huron.

INTRODUCTION

Decades of study of the records of glacial lakes in the Huron and Erie basins, chiefly by those devoted students, Frank Leverett and Frank B. Taylor, have unraveled a complicated succession of changing levels and shifting outlets, a history discussed briefly in every current American text on historical geology. Two contrasted causes produced these changing lake levels in front of the shrinking glacial lobes. One was change in location of the ice front: retreats opening new lower outlets and thus lowering lake levels, or readvances closing outlets and producing higher levels determined by the next-lowest available discharge routes. The other cause, yielding only a series of consecutive lowerings, was deepening of outlets in use. The problem attacked in this paper is the occurrence of pauses in such deepening, the results of which were stationary lake levels and definite shore lines. These pauses alternated with times of active outlet erosion and sufficiently rapid lowering of lake levels to leave no recognizable shore features. We are therefore seeking the reason for the second cause.

[1] Manuscript received July 10, 1950.

The Saginaw Basin is a branch or limb of the Huron Basin, projecting southwestward into Michigan from the larger and deeper feature. During the recession of the Wisconsin ice sheet, this basin determined a large subordinate lobe, whose oscillatory shrinkage is recorded by moraines distributed almost the whole distance from the Indiana-Michigan state line northeast to Saginaw Bay. About 50 miles from the head of the bay, retreat began to expose a surface which sloped toward the shrinking ice, and free drainage of meltwater westward was replaced by standing water at the ice front. The outlet used by this lake water was the Grand River channel, which headed on the divide between present Lakes Huron and Michigan and led westward for approximately 100 miles to the contemporary Glacial Lake Chicago in the Lake Michigan Basin.

Seven different shore lines, successively lower toward the east, record seven pauses in the deepening of this channel, while ponded glacial water stood in front of the retreating Saginaw lobe.[2] Why did not the westward-dis-

[2] Only once during this history was there a retreat sufficient to open temporarily another lower outlet, and none of these seven shore lines belongs to that episode.

charging water *continuously* deepen the Grand River outlet channel?

Monograph 53 of the United States Geological Survey (1915) contains most of Leverett and Taylor's matured interpretations of Michigan's Pleistocene geology. Although each later published (1939) revisions, corrections, and additions to the interpretations in the monograph, none of these alters the succession or the relations set forth in the monograph for the Saginaw Basin. The only explanation for the alternate lowerings and still-stands to be found in all their work was Taylor's brief suggestion (Leverett and Taylor, 1915, p. 360) that a stoping process, twice initiated at the western end of Grand River channel by two drops in Lake Chicago's level, progressed eastward up the channel, eventually affecting static water levels in the Saginaw Basin.

SALIENT POINTS IN THE LAKE HISTORIES, FROM LEVERETT AND TAYLOR

Leverett and Taylor grouped the recessional moraines of the Southern Peninsula of Michigan into four systems, all of them represented in the Saginaw lobe deposits. The last two, the Lake Border and Port Huron systems, the only two concerned in this study, are remarkably deployed in the Saginaw Basin into separate moraines describing concentric arcs rudely parallel with the outline of the present bay. Thirteen of these moraines are recognized for the Lake Border system and three for the Port Huron. The first nine of the earlier system were landlaid throughout. The remainder of the sixteen, lying east of the divide, were built partly or wholly in the ponded water whose discharge was westward across the first nine to be deposited.

When the glacial front stood at the Flint (8th) moraine of the Lake Border system, Lake Maumee at the western end of the Erie Basin had abandoned its outlet to the Wabash River at Fort Wayne, Indiana, and was using a newly uncovered, lower spillway northward and westward across the "thumb" of Michigan. This, the Imlay River, skirted the southern edge of the Saginaw lobe and entered the Grand River channel near Maple Rapids, just west of the convex front of the lobe. There was as yet no lake in the Saginaw Basin.

With the retreat from the Flint moraine came the first exposure of eastward-sloping terrain and therefore the first possibility of standing water in the basin. The lake was described as a small, crescent-shaped water body, moraine-bounded along its convex westward side, ice-dammed along its northeast concave margin, and discharging at Maple Rapids into the Grand River channel. According to the interpretation in the monograph, Imlay River presumably abandoned that lower part of its channel now occupied by Hayworth Creek (fig. 1) when ice withdrew from the Flint moraine, and crossed this moraine 10 or 12 miles east of Maple Rapids, entering the lake and adding its volume to the outlet river.

When the ice front readvanced to build the Owosso (9th) moraine, this lake was obliterated, Imlay River was turned back into its abandoned lower course, and, at the apex of the lobe, the new moraine for a few miles of length was piled up on the back of the Flint moraine.

Retreat from the Owosso moraine reestablished Early Lake Saginaw, and a new short-cut for Imlay River was soon discovered across both the Flint and Owosso moraines northwest of the city of Flint. The lake level was held stationary long enough for a shore record to be made, then fell between 10 and 12

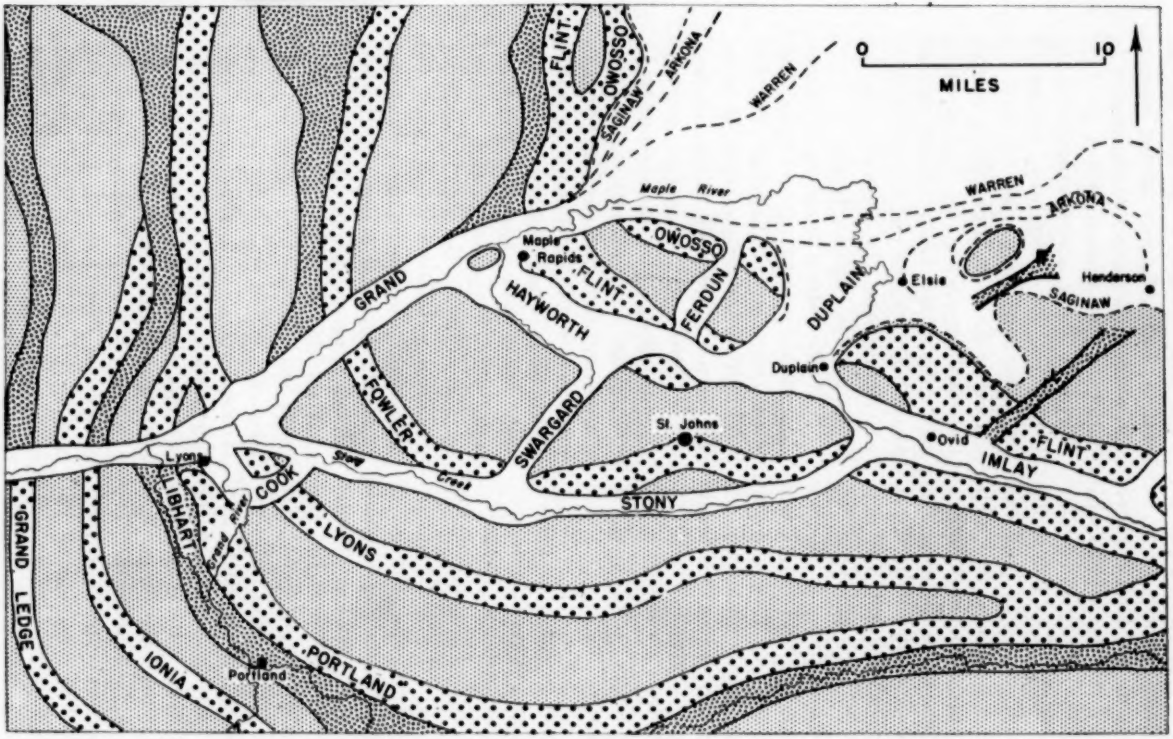

FIG. 1.—Features near the head of Glacial Grand River. After Leverett's 1924 map. The 1915 maps (Leverett and Taylor, 1915, pl. 7, fig.1) showed units of the Owosso moraine about Elsie and a part of the Henderson moraine farther east, where the revision shows only lake bottom and ground moraine. The Lewis and Baker channels have been added.

feet as Glacial Grand River trenched into the Owosso moraine at Maple Rapids, pausing after that fall in level to constitute Second Lake Saginaw.

Four faint water-laid moraines (Henderson, West Haven, Chesaning, and "moraine north of Chesaning")[3] on the lake bottom inside the Owosso appear to record that many more oscillatory shiftings of the ice front before the Lake Border system was complete. The algebraic sum of these oscillations was a general retreat, and during this time lower land became exposed in the "thumb," the re-entrant angle between the Saginaw and Huron lobes. This finally brought about an abrupt drop in Lake Maumee to the level of Second Lake Saginaw, the two lakes becoming one in the same sense that Lakes Michigan and Huron today are one. Abandonment of all the Imlay River channel marked this fusion, which formed Lake Arkona. No change in the Saginaw Basin itself occurred immediately except a broadening of the lake.

But Lake Arkona had three different levels during its history, the total fall of its surface from Second Lake Saginaw's level (with which Arkona's first level coincided) having been about 15 feet. Thus there were two pauses in channel deepening at Maple Rapids during Arkona time, alternating with two times of channel erosion.

The next forward movement of the ice front, recorded by the Port Huron (14th) moraine, overrode the strait in the re-entrant angle and forced the Erie portion of Lake Arkona to rise 30 feet or more to a spillway (Ubly) hard against the ice front at the tip of the re-entrant. As during Maumee-Imlay times, the discharge entered the lake in the Saginaw Basin and went on down Grand River.

This dismembered Erie portion of former Arkona, called Lake Whittlesey, endured long enough for well-developed beaches to form on the landward side. They are presumably contemporaneous with the lowest Arkona beach in the Saginaw Basin. But nowhere have these beaches been found on the frontal slopes of the Port Huron moraine, the marker for the lowest Arkona-Whittlesey ice dam's greatest advance.

The retreat from the Port Huron moraine was greater than any earlier withdrawal of the lobe fronts had been. It cleared the region south of Syracuse, New York, sufficiently to offer the ponded water in Ohio, Michigan, and Ontario an escapeway to the Mohawk lower than the head of the Grand River channel, although that had been trenched into perhaps 30 feet since the days of Early Lake Saginaw.

Lake Wayne, this eastward-draining successor to Whittlesey and lowest Arkona, endured only during the maximum of the retreatal phase following the Port Huron advance.

The next advance, to the Bay City (15th) moraine position, promptly closed the eastern dischargeway and re-established westward drainage along Grand River channel. The new level in the Saginaw Basin, Lake Warren, stood about 5 feet below the lowest Arkona beach, recording that much channel deepening in the Arkona-Warren interval. Lake Warren left two definite shore lines, about 15 feet apart vertically. In places a weaker intermediate Warren shore has been found. During the life of the lake the two remaining members of the Port Huron morainic system—the

[3] Taylor was a bit uncertain in his identification of two of these. They have very faint relief on the lake plain but are accompanied by boulder belts which he interpreted as residual from wave erosion. The spacing intervals between the definite moraines of the region also seemed to require this interpretation of these features.

Bay City and the Tawas (16th)—were built and record two more oscillations of the front of the retreating Saginaw lobe.

Retreat from the Tawas moraine coincided with lowering to the last stage of Lake Warren. The writer will argue, in this paper, that to maintain that level, there theoretically must have been another advance but that the moraine marking its culmination was entirely water-laid and is still submerged in the water of Lake Huron and Georgian Bay.

Retreat from this theoretical moraine reopened the eastern dischargeway, and, from that time on, no closure of this route to the Mohawk by ice advance ever occurred. After Lake Warren's lowest stage, the Grand River channel ceased to be a glacial lake outlet.

INADEQUACIES AND INFELICITIES IN THE PRECEDING INTERPRETATIONS

That a stoping process could occur along the entire length of the Grand River channel, which is wholly in drift, seems difficult to accept. That there was adequate time for developing "grade" in the channel floor is a question. The most pointed objection, however, is that where Lake Chicago experienced two lowerings, of about 20 feet each, the Saginaw Basin lake series records six lowerings, totaling about 50 feet. Taylor's suggestion of a causal relation seems impossible.

Although six moraines were laid down after the first lake's record was made in the Saginaw Basin and there were seven successively lower lake levels with westward discharge, Leverett and Taylor never attempted correlation of either stationary or falling levels with retreats or with readvances of the ice front.

The Wayne and Arkona stages were pointed out as coinciding with two stages of glacial retreat, Whittlesey and Warren, with two readvance climaxes. But Arkona's history is defined, by implication at least, as covering the time of three retreats and readvances, and Warren's as covering two or three. The writer holds that the only lake stage properly connected with the maximum of a retreat was that of Wayne. Had not an eastern outlet been uncovered in this retreat, it would logically have produced only another, still lower, Arkona stage in the following readvance.

Although the Port Huron moraine was recognized as marking the maximum ice advance during the life of Whittlesey and lowest Arkona lakes, the entire lack of their shore lines on that moraine is but vaguely explained. "Either the ice itself formed the shore, or readvance of the ice or outwash or the erosion of streams issuing from the ice obliterated the marks that the waves produced on the face of the new moraine" (Leverett and Taylor, 1915, p. 379).

THESIS OF THIS STUDY

1. Oscillations of the front of the Saginaw, Huron, and Erie lobes caused the oscillations between times of active deepening and of pauses in such deepening of the Grand River outlet channel.

2a. Lake levels remained sufficiently unchanged during glacial advances to permit the making of shore-line records.

2b. Beginning at the maximum of an advance and the deposition of a moraine and continuing during a retreat, the outlet river deepened its channel, and lake levels steadily fell until retreat ceased and another advance began.

3. The cause of this alternating behavior of the outlet river was alternating maximum and minimum volume of discharge from the melting ice. The river volume during a retreat was increased by the total additional melting that produced the recession. When the ice front readvanced into territory previously

abandoned, as much water was denied the outlet river, per mile of advance, as had been supplied in excess during the preceding retreat. The assumption of approximately equal rates of forward and backward shifting of the glacial front is justified. Figure 2 is an attempt to portray graphically the ideas in items 1–3.

4. During a retreat, the maximum volume that caused the channel erosion readily carried away the finer constituents of the drift being trenched into, although a floor of lag cobbles and boulders gradually accumulated and retarded the deepening. When maximum volume ceased to arrive, the velocity of the shrunken discharge was so decreased that the lag material dropped entirely out of transit. It thus constituted an armor which endured until the climatic cycle had come full-turn: until another lake stage had been recorded, another recessional moraine had been built, another retreat had begun, excess volume was again supplied, the velocity of the outlet river had increased, the effectiveness of the armor had ceased, channel deepening was resumed, and the level of ponded water again dropped.

Because the length of the contributing ice front consistently increased during the succession of moraines and lakes here considered, it is argued that each flood stage of Grand River, greater than the preceding one, was capable of disposing of the boulder armor it found.

The volume of discharge from such melting on the Saginaw, Huron, and Erie lobes as just to balance the rate of glacial flow is unknown, the extent and rate of retreat unknown, the thickness of the marginal belt removed during a retreat unknown. The length of the ice front whose meltwater escaped down this river was from 500 to nearly 600 miles. Can reasonable assumptions for the unknown factors allow a test of the quantitative sufficiency of the proposed mechanism?

Taylor was of the opinion that 25 miles was a fair estimate for a retreat from one of these deployed moraines. His figure 6 in the monograph indicates this distance between the Port Huron moraine on the "thumb" and the hypothetical position of the ice margin out in Lake Huron for the following Lake Wayne stage, although he also shows a distance of 60 miles between these markers along the axial line of Saginaw Bay.

Because the Port Huron retreat opened the Syracuse outlet, it was greater than retreats from the later Bay City or Tawas moraines and may logically have been greater than from any earlier ones. If 25 miles was its average extent, the building of other moraines may be assumed to have been followed by at least 15 miles of frontal withdrawal. Such a retreat would remove all glacial ice for from 7,500 to 9,000 square miles. If the thickness of this vanished ice averaged a fifth of a mile, then between 1,500 and 1,800 cubic miles of ice became liquefied in each of these frontal back-steps, and, because none of the lakes was very deep, most of this water disappeared down the Grand River outlet channel.

Another assumption must be made for the duration of this excess melting. The retreat was across a region of low relief and not far from the same latitude as north-central Iowa, where Gwynne (1942) found faint, close-set, "annual" ridgings made during the retreat of the Des Moines lobe at the same time that some of these Saginaw Basin events were occurring. Annual moraines have also been found in Quebec (Norman, 1938), in Sweden (De Geer, 1940), and in Finland (Sauramo, 1918). Spacings range between 300 and 1,000 feet. Gwynne's average was 15 moraine ridgings per mile.

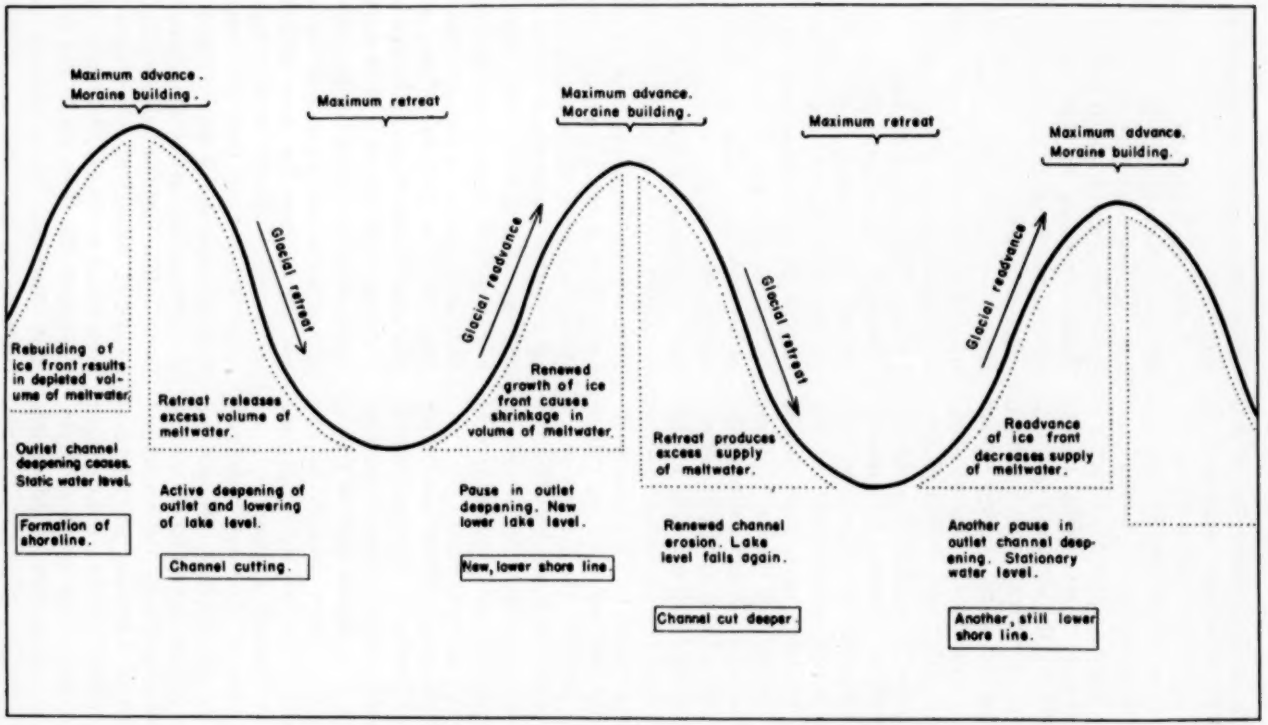
Maximum advance.
Moraine building.
Maximum retreat
Maximum advance.
Moraine building.
Maximum retreat
Maximum advance.
Moraine building.
Glacial retreat
Glacial readvance
Glacial retreat
Glacial readvance
Rebuilding of ice front results in depleted volume of meltwater.
Outlet channel deepening ceases. Static water level.
Formation of shoreline.
Retreat releases excess volume of meltwater.
Active deepening of outlet and lowering of lake level.
Channel cutting.
Renewed growth of ice front causes shrinkage in volume of meltwater.
Pause in outlet deepening. New lower lake level.
New, lower shore line.
Retreat produces excess supply of meltwater.
Renewed channel erosion. Lake level falls again.
Channel cut deeper.
Readvance of ice front decreases supply of meltwater.
Another pause in outlet channel deepening. Stationary water level.
Another, still lower shore line.

FIG. 2.—Depiction of the succession of interrelated events which produced the lake stages in Saginaw Basin

Wilson (1943) estimated from incomplete cores of varved sediment at Sandusky Bay, south side of Lake Erie, that submergence in glacial lake water endured here for at least 12,223 years, this time beginning with Later Lake Maumee and covering but little more than the life of all the subsequent glacial lake stages south of the Whittlesey outlet. During this time the front of the Erie lobe retreated approximately 250 miles, the average rate being 108 feet per year or nearly 50 years per mile of retreat. However, if two-thirds of the territory abandoned in each individual retreat was retrieved during the next advance, then the ice edge itself shifted through five times the distance actually lost. No matter how many or how few the oscillations that occurred during this 250 miles of total withdrawal of the glacial front, the actual distance traversed in its shiftings was, by our assumption, 1,250 miles. Hence the actual rate of shifting was 1 mile in about ten years, or nearly 550 feet per year, a figure which agrees well with the rates obtained from the annual moraines.[4]

[4] The rate of glacial retreat in New England has been computed (Antevs, 1922) from the varve record left in numerous small glacial lakes as the ice front withdrew through a distance of 185 miles. The total time required was 4,100 years. The rates for different parts of the distance varied, but the average was 24 feet annually. Thus it required nearly 220 years for 1 mile of average retreat. This computation, however, includes time elapsed during whatever readvances occurred but are not in the record. If each oscillatory advance repossessed two-thirds of the area just abandoned, the edge of the New England ice must have melted back and readvanced, on the average, at five times the above rate. This would be 120 feet annually, or 1 mile in 44 years, a considerably slower rate than that in Iowa or the Great Lakes. The marked differences in topography should be one cause for the difference in rate.

Sauramo (1923, p. 142), using the varve record of southern Finland, obtained retreatal rates of 10.8 and 13.2 years per mile during his Periods 2 and 3. Period 4 saw a greatly accelerated rate because of marked decrease in mass of the wasting ice, the marginal recession for 75 miles averaging 1 mile in 8 years.

Accepting the interpretation of annual moraines and using the slower rate, a 15-mile retreat would have been consummated in 264 years. This would mean that 6⅔ cubic miles of ice, in excess of the "normal" wastage, would be melted annually during the retreat, which would yield more than 29,500 second-feet additional discharge to the outlet river. When the following advance occurred, the outlet river's "normal" volume would be shrunken by an equivalent amount (although for a shorter time) because the vanished ice must be replaced on a belt about two-thirds as wide as that exposed previously. Thus, by the figures used above, the maximum discharge down Grand River exceeded its minimum by nearly 60,000 second-feet. This would be comparable to adding, during a retreat, more than the average flow of the Wabash River and to subtracting that quantity during an advance.[5]

The quantitative sufficiency of the mechanism advocated may be challenged on another point. Was 264 years long enough for the outlet to be deepened from 5 to 10 feet? We have no knowledge of the volume of meltwater that balanced glacial flow, hence no estimate for the percentage increase or decrease produced in Grand River by the glacial oscillations. But the erosional ability of a stream drops in geometrical ratio as volume decreases, and a grant of more time involves a correspondingly smaller volume per year.

It is conceivable that the recently deposited drift crossed by the outlet river farther west still retained much interstitial water and lacked the degree of

[5] A considerably more rapid melting-down of a portion of the Scandinavian ice sheet is reported by Mannerfelt (1945). If the rate of thinning that he computed (3½–4 meters yearly) obtained on the Saginaw lobe, the duration of a retreatal stage would scarcely exceed 150 years, and the Grand River discharge would be correspondingly increased.

resistance to the river that later compaction has given it. The erosion that drew down the lake levels did occur, and so did the pauses in outlet erosion.

FIELD EVIDENCE SUPPORTING THE THESIS

Monographic treatment may imply, but does not guarantee, intensive and detailed study. Moreover, the Leverett and Taylor investigations were made in horse-and-buggy days, without much help from road cuts or gravel-pit sections and largely without benefit of topographic maps. Thus a week's field work in 1950 along the southern side and convex front of the Saginaw lobe revealed several significant items either not mentioned in monograph 53 or at variance with descriptions and interpretations therein. The four most important items are briefly as follows:

1. A well-developed valley train enters Imlay River channel from the Owosso moraine and is readily traceable downstream along Hayworth Creek as far as Maple Rapids. It was deposited at the Owosso (9th) maximum in an Imlay channel of Flint age and became much eroded by the last of Imlay discharge during Henderson (10th moraine) time. There is no trace of such a deposit in the Imlay channel upstream from the city of Owosso.

2. Imlay River never entered Lake Saginaw near Duplain, as Taylor thought it did (Leverett and Taylor, 1915, pp. 257–258). Early Lake Saginaw and its immediate successor, Earliest Lake Arkona (= Later Lake Saginaw) discharged across the Flint moraine by three contemporaneously functioning spillways: Duplain, Ferdun, and Maple Rapids (see fig. 1). The Duplain spillway led directly into Imlay River, which, throughout the entire history shown in fig. 1, discharged down Hayworth Creek Valley.

3. There are five glacial river channels unnoted in the monograph but significantly related to those described therein, the existence of which modifies and makes possible more precise interpretations of the region's glacial history.

4. An older drift sheet, very bouldery in both its till and its washed phases, has been reached in the deeper glacial and postglacial river channels, and its contributions have played a part in the later glacial drainage history.

THE OWOSSO VALLEY TRAIN

Taylor's idea that the Owosso ice locally crowded up on the back slope of the Flint moraine near Maple Rapids is supported by features of the Flint's northern slope near Ovid and Owosso. For several miles the frontal ridge of the Owosso moraine (which ridge Taylor considered a part of the Flint) closely conforms to the sinuous course of the Flint, and the two are separated only by imperfectly developed marginal or submarginal channels. This ridge is really only a line of partially separated, elongated, kamey and eskerine tracts, from whose juxtaposed ice a strong flow at Owosso maximum crossed the Flint moraine at Ovid, at Lewis Drain, and at Owosso. Another kamey, gravel-floored discharge from Owosso ice, the Caledonia channel, entered the marginal drainage route at Corunna and crossed the Flint by the Owosso transection. All told, about 20 miles of Owosso ice front appears to have poured its meltwater across the Flint at these three places. At no other frontal stand along the Saginaw lobe's southern limb in this latitude is there record of any comparable meltwater discharge or load of gravel.

Along the front of the Flint flowed the early Imlay River. Its volume and gradient were inadequate for carrying away all the load contributed by the Owosso discharge. The valley train that resulted has been traced from Owosso to Maple Rapids, 30 miles farther west, its surface descending about 25 feet in that distance. Numerous large gravel pits show a deposit at least 30 feet in maximum thickness. The material is well washed and well sorted, has many strata of westward-dipping foresets, and is progressively finer-textured with increasing distance from the moraine. It is to be sharply distinguished from contiguous morainal gravel deposits.

When ice withdrew from the Owosso moraine, the valley train was promptly attacked by the now underloaded Imlay River, leaving only terraces and mid-channel hillocks as the glacial riverway was re-excavated. Only the time between Owosso and Henderson maxima is allowable for this destruction because Imlay River, finding a lower spillway northwestward from Flint, Michigan, when the Henderson ice front retreated, entered Lake Saginaw at Flushing and abandoned the entire channel length that we are considering in this study.

IMLAY RIVER'S RELATIONS TO LAKE SAGINAW

Taylor's idea that Imlay River ever entered Lake Saginaw before the above-noted spillway was uncovered is in error but, without today's maps and excavations, would be almost impossible to disprove. Taylor located his earlier debouchment in the Duplain gap. Here the well-marked Imlay channel, now followed by Maple River, virtually disappears as it encounters the broad embayment of the low, sandy lake plain flanked by Lake Saginaw beaches. Here is the head of Hayworth channel, correctly interpreted by Taylor as a part of the early Imlay River course, its floor only 5 feet higher than that of most of the embayment bottom but 15 feet lower than a beach ridge margining the bottom land only a mile distant. Here the Owosso valley train continues from the Maple-Imlay channel westward across the head of the embayment and on down the Hayworth-Imlay channel. Not a pebble from the 30 feet of gravel here was ever carried into Lake Saginaw, either when the fill was made or later when 20 feet or more of dissection by Lake Maumee discharge occurred.[6]

Owosso ice covered much of the Duplain sag when the valley train was deposited, and Imlay River had no choice but to follow the Hayworth channel of Flint moraine time. But only ponded water stood here during the dissection, and only southward discharge from the head of the embayment can explain the absence of a deltaic deposit in it.

A remnant of the Owosso valley train, covering about 2 square miles, lies on the south side of Imlay channel at the head of Hayworth and the junction of Duplain and Maple-Imlay. Taylor was puzzled by its features, describing it (Leverett and Taylor, 1915, p. 257) as "more like a kame than any other type—a kame deposited in a small re-entrant of the [Flint] ice front and shaped partly by the ice with which it was in contact and perhaps partly by the later [Imlay] river current." Today the 5-foot contouring of the Perrinton and Elsie topographic maps (see fig. 3) and the existence of two large gravel pits make clear that the lower southern part of the triangular area is a shallow channel of the aggradation episode. Its two northerly margins are walls of deeper channels, one from Maple-Imlay and one from Lake Saginaw along the Duplain sag. The highest part, in the apex of the triangle and along the western side, Taylor described as "a prominent gravel ridge—like a parapet—[which] closely resembles a beach ridge." It is, in fact, two gravel beaches, the higher at 735+ feet A.T., built of the local gravel by Lake Saginaw waves after the episode of valley-train dissection was completed, i.e., in early post-Henderson time.

Another conspicuous remnant of the Owosso valley train is a mid-channel hill about a mile long, separating Maple and Little Maple rivers where both are using the Imlay channel west of Ovid. The summit of the hill is about 25 feet above the late Imlay floor on the north side.[7]

PREVIOUSLY UNNOTED GLACIAL RIVER CHANNELS

The glacial river channels involved in this problem and recognized in monograph 53 are the Grand, the Lookingglass-Libhart, the

[6] The reconnaissance character of Leverett and Taylor's study in this region is shown by the existence of an esker associated with the Ferdun channel, higher, longer, and more massive than any described under "Moraines of the Eastern Limb of the Saginaw Lobe" (Leverett and Taylor, 1915, pp. 249–250). It is visible for 3 miles in any direction. The failure to recognize its existence, or that of the channel, in the monograph is a further indication that neither author ever traversed the township in which these features are subcentrally located. Both esker and channel show on the Perrinton topographic map (surveyed 1915–1916). Leverett's 1924 depiction of a narrow lake embayment here seems obviously only a map interpretation.

[7] A medial gravel ridge occupies the last mile of Maple-Imlay south and southwest of Duplain, with the northwest-flowing Maple River on one side and a shallower, southeast-draining depression on the other. If the ridge is a mid-channel remnant of the valley train, the flanking trenches should both open out into the Duplain sag. But the shallower one is blocked in the village by a transverse sand ridge whose crest is above 735 feet A.T. It is the original "Duplain beach" of Taylor (1897), made by the highest recorded stage of Lake Saginaw. The blocked channel seems to be a survival from channel braids on the valley-train surface, similar to the one south of the "parapet" beach.

Maple Rapids, the Stony, the Imlay (Maple and Hayworth), the Duplain, and the Corunna-Owosso. The present interpretation of the Duplain reverses its supposed direction of flow. The Corunna-Owosso channel is reinterpreted as a discharge route from ice on the Owosso moraine, not a detour or accessory channel of Imlay River.

The Swargard channel was added on Leverett's 1924 map. The writer here adds the Cook, the Ferdun, the Lewis, the Baker, and the Caledonia channels, all a part of the shifting drainage changes in front of the shrinking Saginaw lobe.

Lyons. The modern Grand now follows that course.

Retreat from the Lyons (6th) moraine allowed the Cook channel to form, the swampy col in it but little above 710 feet A.T. and the actual channel floor probably still lower beneath the swamp accumulation. Cook was but a small distributary of Stony channel.

Stony channel carried marginal drainage along several miles of the Fowler (7th) moraine front. A low place between the Lyons and Fowler moraines in the vicinity of Pewamo was aggraded to make a broad gravel flat at 720 feet A.T. Then, the load apparently decreasing, the

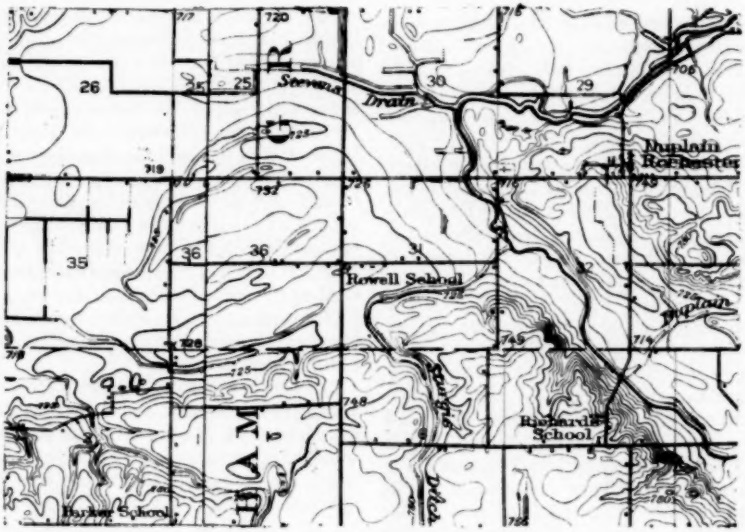

FIG. 3.—Vicinity of Duplain. From Elsie and Perrinton topographic maps, U.S. Geological Survey

During the building of the Portland (5th) moraine, marginal drainage from the southern side of the Saginaw lobe used an imperfectly developed channel now followed by Lookingglass River as far as Portland, thence by modern Grand River for 3 miles, and thence by Libhart Creek to its entrance into the Glacial Grand. The floor of Libhart channel and terraces at and near junction with Glacial Grand (Portland topographic map) record a main channel floor but little below 740 feet A.T. at that time.

Retreat from the Portland moraine exposed a surface lower than the Libhart floor, and consequently the Glacial Lookingglass crossed to the back of the moraine about 4 miles south of Glacial Stony trenched into its own deposits. The Cook distributary probably was still functioning, for its floor is lower than the Pewamo flats.

Glacial drainage along the full length of Stony channel ceased on retreat from the Fowler moraine. Despite a distributary-like relation in ground plan, Stony almost surely never was supplied from Imlay River. The eastern few miles of its length are very weakly developed, compared with the rest of its length, and at its junction with the Imlay its floor hangs about 20 feet above that of the main channel. Except for later local erosion, there is hardly a notching of the top of the Imlay bluff at the junction. On the

monograph map, plate VII, Stony was shown as the lower part of a channel 50 miles long, called the "Bennington." The 1924 map, issued after two of the topographic maps covering this Bennington channel had appeared and surveys had been initiated on two others, omitted 35 miles of this supposed channel and depicted Stony correctly, as in the accompanying figure 1. This change appears to be another revision from map study.

Cook and Stony are of interest here because they record a deepening in the trunk drainage line, the Grand River channel, of at least 20 feet from the time of the Portland (5th) maximum to that of the Fowler (7th).

The Swargard channel records a broad, shallow overflow across the Fowler from the earliest glacial drainage along the front of the Flint (8th) moraine. Swargard may have been an Imlay distributary before the trenching which followed accession of Lake Maumee's overflow. Although Swargard's junction with Stony is accordant, the trenching in Pewamo flats and Cook channel seems best explained as done by Swargard discharge. So interpreted, the Grand River channel floor had been lowered another 10 feet by Flint moraine time.

Imlay's course in the Saginaw Basin was along an intermorainic depression into which the river trenched to maximum known depths of 30 feet. Following this came the deposition of the Owosso valley train, nearly filling this trench and spreading a broad sheet of gravel in lower Hayworth, where lay a deeper portion of the original depression. Flat-topped remnants of this valley train, left from the immediately post-Owosso dissection, are 720–725 feet A.T. at Maple Rapids, and the floor of Glacial Grand at this place thus was approximately 715 feet A.T. when the Owosso moraine was deposited.

Imlay River may not have deepened the Glacial Grand channel very much before the making of the Owosso valley train, for all the debris from trenching along the front of the Flint had to be carried across the state to the Allendale delta in Lake Chicago, west of Grand Rapids.

Taylor depicted a broad gap in the Flint moraine at Maple Rapids, into which the Owosso ice pushed and built its own moraine. He believed the gap to have been eroded by the outlet of a pre-Owosso lake, across the newly built Flint moraine. This gap, however, is at least four times as wide as that made in the Owosso moraine at the same place by all subsequent outlet erosion. It seems much more probable that the pre-Owosso gap was largely a sag in the crest of the Flint moraine, a discontinuity similar to the Duplain sag in both Flint and Owosso moraines and to others in virtually every moraine the region possesses.[8]

Early Lake Saginaw, the first of the lake series in the Saginaw Basin to leave positive records, was dammed by the Owosso moraine. Because some of its beaches lie on the back slope of the Henderson moraine at the same altitude, 735+ feet A.T.,[9] as at Duplain, this highest recorded lake stage must have endured without lowering by outlet erosion during Owosso retreat, Henderson advance, and much, if not all, of Henderson retreat. Its waves made the "parapet" beach ridge on the triangular remnant of the Owosso valley train 3 miles west of Duplain; hence dissection of the valley train was already advanced.

This duration through two retreats is a seeming contradiction of the thesis that retreating glacial episodes were outlet-deepening, and therefore lake-lowering, times. The contradiction disappears when one remembers that the Owosso valley train had nearly filled the early Imlay channel and had extended an unknown distance down the Glacial Grand. The energies of late Imlay River and the contemporaneous Glacial Grand were all absorbed in the immediate task of restoring the earlier channel depths.

Ferdun channel's southern part was originally an esker trough across the Flint moraine. As a surface route across both Flint and Owosso moraines, it was one of three dischargeways for Lake Saginaw, Maple Rapids and Duplain being contemporaries. Because its floor across the Flint is accordant with that of the late Imlay trench cut into the valley train in Hayworth, Ferdun is known to have functioned at least to

[8] One has only to compare the map in the monograph with the revision issued in 1924 to see what difficulties in moraine mapping these discontinuities constitute. At the front and along the south limb of the Saginaw lobe, Leverett in 1924 abandoned more than 100 miles of moraine lengths that were shown in 1915, bodily shifted nearly 100 more miles, and added 90 miles of newly depicted linear moraines! The moraine least altered was the strong Flint, with the Imlay channel hard against the foot of its southern slope.

[9] Altitudes are of beach crests, taken from the topographic maps. Taylor's figure was "720 to 725 feet aneroid," doubtless measured at the lakeward base of a ridge. The maps show that 730 is a better figure.

the end of that episode of dissection, at which time Lake Saginaw had been lowered to 710–715 feet A.T. (=Highest Arkona) and Imlay had found the Flint-Flushing short cut.

At this same Later Lake Saginaw time, Duplain discharge was recorded by the 715-foot col at the head of Hayworth, essentially the channel in the valley train which Imlay abandoned for the route north to Flushing.

Part of the Lewis channel transects the Flint moraine and part appears to be an esker trough in the Owosso. Lewis contributed considerable gravel to Imlay during the valley-train aggradation. Glacial drainage continued to use it during Henderson maximum, when the valley train was still being trenched and Lake Saginaw still stood at its highest level.

The Baker channel crosses an Owosso moraine unit along what originally must have been another subglacial drainage route. It appears to have been supplied later from Henderson meltwater and to have found Lake Saginaw still at 725–730 feet A.T.

From this examination of sequences among glacial drainage channels of the region, we conclude that the earliest ponded water back of the Owosso moraine had three outlets—Duplain, Ferdun, and Maple Rapids—to the same trunk stream—Glacial Grand River—and that, up to the time of making the Later Lake Saginaw beach, none obtained precedence over the others. Only after the Owosso valley train had been eroded could Grand River, fed by lake water alone, begin deepening when a glacial retreat was in progress and offer opportunity for competition among the three. This was during the West Haven retreat, next after the Henderson withdrawal. But the three ran neck and neck in that first deepening. Then Duplain and Ferdun fell out of the contest, and only Maple Rapids outlet continued to be deepened when the Chesaning retreat occurred and the ponded water fell to the intermediate Arkona level.

THE SUBJACENT BOULDERY DRIFT

Remnants of a channel-bottom boulder armor, suggesting that which figures in the writer's proposed mechanism, were reported by Taylor to lie on "terraces and boulder benches" from the general vicinity of Maple Rapids as far down the Grand River outlet channel as Lyons. Referring to the entire deepening, Taylor said that "with the floor of the channel heavily protected by boulders, the lowering was slow and required a long time." But it was also spasmodic, with three times as many episodes when the armor was ineffective as Taylor's explanation (stoping initiated by Lake Chicago lowerings) can account for.

Because the supply of boulders adequate for checking channel erosion during a glacial advance is assumed in this study to have been swept away by the enlarged and quickened Glacial Grand River during the next retreatal episode, the boulder armor now lying on the late Warren channel floor (Leverett and Taylor, 1915, fig. 2) is not the cumulative result of the earlier episodes of deepening. Its impressiveness is due largely to contributions from a subjacent bouldery drift that was reached only in this latest channel deepening. The channel floors along Stony and Hayworth dischargeways, although earlier developed, shorter lived, and less deeply trenched, show the same extreme boulderiness in their downstream portions but, traced upstream, possess only boulderless and almost cobbleless gravel.

Taken with the exposures in ravines and pits near Maple Rapids of bouldery till and outwash beneath the moraine drift and its gravel, this seems good evidence that these excessively bouldery channel floors are not cumulative aggregates. The high percentage of glacially marked boulders in these exposures also argues for only a brief experience at the bottom of the outlet river.

LACK OF SHORE LINES ON THE PORT HURON MORAINE FRONT

The glacial front at which the Port Huron moraine was built served also as the dam for Whittlesey and Late Arkona lakes. But shore lines recording these contemporaneous lake levels have not been found on this moraine. Taylor reported both shores on older moraines in several places where they, with the Port Huron, outlined narrow bays at these lake levels. He reasoned that their strong development in such constricted situations was impossible and that the open expanses of water required for adequate waves existed during the immediately pre–Port Huron retreat and were destroyed as the Port Huron advance culminated.

However, some other explanation is needed for the lack of these shores in places where the Port Huron moraine faced wide stretches of lake water. The thesis of this paper supplies a satisfactory explanation. It requires that a static lake level exist only during a glacial advance and that this level start dropping immediately when a retreat begins. No moraine fronts, therefore, should carry more than the faintest shore lines of their contemporaneous lakes.

SUMMARY

Glacial Grand River, flowing west across the southern peninsula of Michigan, carried meltwater directly from the Saginaw glacial lobe during the building of the Lake Border and Port Huron moraine systems and from the successive glacial lakes impounded in front of this lobe. It also received the overflow of several successive lakes dammed by the Erie and Huron lobes.

Because the named lakes were largely a succession of lowered levels in the same basins, discharging through the same outlet, some mechanism must have operated to produce the sequence of static levels alternating with lowering levels.

The volume of this great glacial river varied between maximal discharge, when the contributing lobe fronts were retreating, and minimal discharge, when their fronts readvanced. Because the successive lowerings resulted directly from outlet deepening, the mechanism proposed is (1) that the greater volume and velocity of the river during a glacial retreat produced the episodes of lowering levels and (2) that during a readvance the weakened river, hampered by a bottom armor of lag boulders and unable to deepen its channel, allowed the static levels time for making shore-line records.

The pondings began when the shrinking Saginaw lobe had retreated east of the Michigan-Huron divide. Each moraine thereafter deposited should theoretically be matched by a lake stage. But the earliest of these lake levels, Early Lake Saginaw, endured through the retreat from the Owosso moraine, the advance to the Henderson moraine, the retreat from that moraine, and (theoretically) the advance to the West Haven moraine.

This departure from the theoretical schedule resulted from a literal clogging of the outlet river by a considerable valley train of Owosso age, brought into Glacial Grand River below the lake outlets. Low places in the Owosso moraine provided three outlets, all continuing to function until the valley train had been sufficiently cleared away. After that, deepening in one outlet outdistanced the other two, and all subsequent lake discharge from the Saginaw Basin used and continued to deepen the Maple Rapids outlet channel.

According to the scheme proposed, Early Lake Saginaw endured from the retreat from the Owosso moraine to the building of the West Haven moraine. Early Lake Arkona (=Later Lake Saginaw) endured during the advance which culminated in the Chesaning moraine. Intermediate Arkona existed while the lobe front was advancing to the position of the moraine north of the Chesaning. Late Arkona's existence coincided with the advance to and the building of the Port Huron moraine. Early Lake Warren levels were recorded while the ice front was advancing to the Bay City moraine's position. Intermediate Warren endured through the episode of advance to the Tawas moraine. Lowering to the Late Warren stage coincided with retreat from the Tawas, the last moraine left on

land in extreme eastern Michigan. To maintain that lowest Warren level while its shore lines were made, theory requires another glacial advance. The moraine marking its culmination is assumed to have been entirely water-laid and still to be submerged in Lake Huron and Georgian Bay.

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LIQUIDUS TEMPERATURES IN MIXTURES OF THE FELDSPARS OF SODA, POTASH, AND LIME[1]

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ABSTRACT

Liquidus data are reported for a series of forty-two ternary feldspar compositions, including compositions at each 10 per cent interval by weight of soda, potash, and lime feldspars. Only two fields, those of feldspar and leucite, appear on the liquidus surface. In addition to the liquidus data, the temperature of appearance of a second solid phase was determined for some of the compositions. Measurements of the index of refraction of the feldspar glasses are recorded. These data are principally of value as a starting point for studies of these same compositions with water as an additional component.

INTRODUCTION

The various feldspars are among the most common rock-forming minerals. Naturally, petrologists and mineralogists have devoted considerable time to studies of the properties, occurrence, and interrelations of feldspars, particularly those of soda, potash, and lime. One of the first studies made by Day and Allen (1905), at the U.S. Geological Survey just previous to the founding of the Geophysical Laboratory, was concerned with the melting relations of the feldspars, and today, even though nearly a half-century has elapsed and many studies have been made, there is still much to be learned about their stability relations.

THE INDIVIDUAL FELDSPARS

The melting relations of each of the feldspars of lime, soda, and potash have been determined. Day and Allen (1905) found that lime feldspar melted congruently. Rankin and Wright (1915) in a study of the system CaO-Al_2O_3-SiO_2 showed its melting relations in the presence of excess of each of its components. Greig and Barth (1938) determined the congruent melting temperature of soda feldspar and showed its melting relations with the feldspathoid nepheline. Schairer and Bowen (1947*a*), in a study of the system Na_2O-Al_2O_3-SiO_2, showed the melting relations of soda feldspar in the presence of excess of each of its components. Morey and Bowen (1922) established the incongruent nature of the melting of potash feldspar to crystals of leucite and a liquid more siliceous than feldspar. Schairer and Bowen (1947*a*), in a study of the system K_2O-Al_2O_3-SiO_2, showed the melting relations of potash feldspar in the presence of excess of each of its components.

PAIRS OF FELDSPARS

The melting relations of each pair of the feldspars under consideration have been determined.

SODA FELDSPAR–LIME FELDSPAR

Bowen (1913) studied the plagioclase series and found that at high temperatures soda and lime feldspars form a complete and unbroken series of solid solutions with a continuous rise in the liquidus and solidus from pure soda feldspar to pure lime feldspar. Schairer (unpublished) in a study of the system plagioclase-silica prepared feldspar composi-

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tions at each 10 per cent interval by weight between soda feldspar and lime feldspar and one additional feldspar (soda feldspar 95, lime feldspar 5). Liquidus data on these agree perfectly with the data of Bowen on the plagioclases, and all data at and above the temperatures of beginning of melting indicate a complete and unbroken series of solid solutions. Tuttle and Bowen (1950) found two different crystalline modifications of soda feldspar with an inversion at about 700° C. and a probable break in the plagioclase series at temperatures well below that at which a liquid phase appears in the dry binary system. They are continuing their studies of these feldspar relations.

SODA FELDSPAR–POTASH FELDSPAR

Schairer and Bowen (1935) published a preliminary paper on the system $NaAlSiO_4$-$KAlSiO_4$-SiO_2 and gave a preliminary diagram for the join $NaAlSi_3O_8$-$KAlSi_3O_8$. Recently Schairer (1950) published the data for this join and the revised diagram. There is a complete series of solid solutions between these two feldspars, with a minimum on the melting and freezing curves and a further complication that all these feldspars with more than 49 per cent potash feldspar melt incongruently to leucite and liquid. A short time ago Bowen and Tuttle (1950) published on the system $NaAlSi_3O_8$-$KAlSi_3O_8$-H_2O. They showed (1) that the complete series of solid solutions between these two feldspars persists in the presence of water at low and moderate pressures; (2) that, as the pressure of water is increased, the incongruent melting of potash feldspar and potash-rich feldspars is eliminated; (3) that the position of the minimum on the melting and freezing curves shifts from $65NaAlSi_3O_8$ $35KAlSi_3O_8$ in the dry system toward compositions richer in soda feldspar; and (4) that the solid solutions of $KAlSi_3O_8$ and $NaAlSi_3O_8$, although complete at higher temperatures, are only partial at lower temperatures and the feldspars prepared at higher temperatures unmix or, conversely, a pair of feldspars (one soda-rich and the other potash-rich) stable at lower temperatures become one homogeneous feldspar at the higher temperatures. Additional relations between the low-temperature and high-temperature crystalline forms of these pure feldspars and solid solutions are now being studied.

LIME FELDSPAR–POTASH FELDSPAR

Schairer and Bowen (1947*b*), in a study of the system leucite-anorthite-silica, determined the relations between lime feldspar and potash feldspar. All compositions between lime feldspar and potash feldspar become completely crystalline only at and below 1,040° ± 20° and then consist of a mixture of pure (or very nearly pure) lime feldspar and potash feldspar. At temperatures above 1,040° ± 20°, leucite appears as one of the solid phases because of the incongruent nature of the melting of potash feldspar.

THREE-COMPONENT FELDSPARS

We have prepared a series of synthetic feldspar compositions at 10 per cent intervals by weight of soda, potash, and lime feldspars with a few additional compositions. All these compositions have been crystallized completely or nearly completely. We have completed and record here our determinations on the liquidus surface. Studies of the beginning of melting are still in progress. All these feldspar compositions will be studied with water as an additional component at a series of isobars.

PREPARATION OF COMPOSITIONS

In the preparation of silicate compositions containing alkalies, special care must be exercised because of volatilization of the alkalies and consequent departure from composition. Particular care is necessary when both Na_2O and K_2O are present. In the preparation of the feldspar compositions for this study, two separate glasses were made with the proportions $Na_2O \cdot 6SiO_2$ and $K_2O \cdot 6SiO_2$. These were prepared by melting Na_2CO_3 or $KHCO_3$, respectively, with purified quartz in platinum crucibles. The loss of alkalies was minimized by first sintering at low temperatures (around 700°) and then gradually raising the temperature to about 1,300° or 1,400°, respectively. The small loss (nearly zero in the case of the $Na_2O \cdot 6SiO_2$ and usually less than 0.0100 g. in the case of $K_2O \cdot 6SiO_2$ on 20.0000-g. samples) was determined by weighing and was corrected by adding the appropriate amount of Na_2CO_3 or $KHCO_3$, respectively, before subsequent fusions. At least three fusions with intermediate crushings of the quenched glasses were necessary to obtain a homogeneous product, which was then crystallized completely to obtain a product which remains dry in a desiccator over fused KOH sticks and is easy to weigh.

Calculated amounts of "$Na_2O \cdot 6SiO_2$," "$K_2O \cdot 6SiO_2$," pure $CaCO_3$, pure Al_2O_3, and purified quartz were weighed into platinum crucibles and fused to prepare the feldspar compositions. Depending on the compositions and consequent viscosity of the glasses, at least four fusions and sometimes as many as eight, with intermediate crushing of the quenched glasses, were necessary to effect complete solution of the Al_2O_3 and secure a homogeneous glass. The homogeneity of each of the glasses was carefully checked under the petrographic microscope by obtaining the index of refraction of the glass grains. Each of the feldspar glasses was crystallized completely or nearly completely at appropriate temperatures (1,000° for most of them) before proceeding with quenching experiments.

DETERMINATION OF LIQUIDUS TEMPERATURES

A determination of the liquidus temperature was made for each of the forty-two ternary feldspar compositions prepared, and for some of the compositions the temperature of appearance of a second solid phase was determined. These studies were made by the method of quenching (Shepherd, Rankin, and Wright, 1909), which consists in holding a small charge of a given composition at a measured temperature for a period of time adequate for the attainment of equilibrium between crystals and liquid, and then chilling instantly to room temperature. Quenched charges were examined with a petrographic microscope to determine the nature and number of crystalline and liquid phases, and crystals were identified by their optical properties. Liquids quench to a glass.

Equilibrium between crystals and liquid in the ternary feldspar compositions was reached in a few hours at temperatures near and above 1,450°, in 1 day at temperatures between 1,350° and 1,450°, and in 2 days at temperatures between 1,200° and 1,350°. Below 1,200° equilibrium between crystals and liquid was attained only after a week or longer, depending on the particular composition.

Quenching experiments were conducted in platinum-wound electric furnaces whose temperature was controlled ($\pm$ 2°) with a temperature regulator—

TABLE 1

RESULTS OF QUENCHING EXPERIMENTS

REFRACTIVE INDEX OF GLASS ±0.003	COMPOSITION IN WEIGHT PER CENT			TEMP. (° C)	TIME	PHASES
	$KAlSi_2O_6$	$NaAlSi_2O_6$	$CaAl_2Si_2O_8$			
	Points in the Feldspar Field					
1.554	10	10	80	1,510	3 hr.	Very rare feldspar in glass
				1,515	3 hr.	All glass
1.548	10	20	70	1,490	7 hr.	Lots of feldspar in glass
				1,495	6 hr.	All glass
1.544	20	10	70	1,490	3 hr.	Small amount feldspar in glass
				1,495	6 hr.	All glass
1.545	10	30	60	1,470	5 hr.	Moderate amount feldspar in glass
				1,475	7 hr.	All glass
1.536	20	20	60	1,470	7 hr.	Moderate amount feldspar in glass
				1,475	7 hr.	All glass
1.536	30	10	60	1,465	7 hr.	Lots of feldspar in glass
				1,470	7 hr.	All glass
1.531	10	40	50	1,440	24 hr.	Moderate amount feldspar in glass
				1,445	24 hr.	All glass
1.531	20	30	50	1,445	3 hr.	Rare feldspar in glass
				1,450	3 hr.	All glass
1.531	30	20	50	1,440	7 hr.	Moderate amount feldspar in glass
				1,445	3 hr.	All glass
1.531	40	10	50	1,440	7 hr.	Moderate amount feldspar in glass
				1,445	3 hr.	All glass
1.521	10	50	40	1,415	24 hr.	Rare feldspar in glass
				1,420	24 hr.	All glass
1.522	20	40	40	1,415	6 hr.	Moderate amount feldspar in glass
				1,420	5 hr.	All glass
1.521	30	30	40	1,415	6 hr.	Very rare feldspar in glass
				1,420	5 hr.	All glass
1.519	40	20	40	1,410	24 hr.	Moderate amount feldspar in glass
				1,415	6 hr.	All glass
1.519	50	10	40	1,405	7 hr.	Moderate amount feldspar in glass
				1,410	24 hr.	All glass
1.517	10	60	30	1,380	24 hr.	Rare feldspar in glass
				1,385	48 hr.	All glass
1.513	20	50	30	1,380	48 hr.	Small amount feldspar in glass
				1,385	24 hr.	All glass
1.512	30	40	30	1,380	48 hr.	Very rare feldspar in glass
				1,385	24 hr.	All glass
1.512	40	30	30	1,370	24 hr.	Small amount feldspar in glass
				1,375	24 hr.	All glass
				1,205	4 days	Lots of feldspar and rare leucite in glass
				1,210	4 days	Lots of feldspar in glass
1.511	50	20	30	1,270	48 hr.	Lots of feldspar and small amount leucite in glass
				1,275	24 hr.	Lots of feldspar in glass
				1,365	7 hr.	Rare feldspar in glass
				1,370	7 hr.	All glass
1.511	60	10	30	1,315	24 hr.	Lots of feldspar and rare leucite in glass
				1,320	24 hr.	Lots of feldspar in glass
				1,365	7 hr.	Very rare feldspar in glass
				1,370	7 hr.	All glass
1.505	10	70	20	1,335	4 days	Rare feldspar in glass
				1,340	48 hr.	All glass
1.504	20	60	20	1,330	4 days	Small amount feldspar in glass
				1,335	72 hr.	All glass
1.505	30	50	20	1,330	4 days	Rare feldspar in glass
				1,335	72 hr.	All glass
1.505	40	40	20	1,315	24 hr.	Moderate amount feldspar in glass
				1,320	24 hr.	All glass
1.504	50	30	20	1,220	72 hr.	Lots of feldspar and very rare leucite in glass
				1,225	4 days	Lots of feldspar in glass
				1,310	27 hr.	Rare feldspar in glass
				1,315	24 hr.	All glass

TABLE 1—*Continued*

Refractive Index of Glass ±0.003	Composition in Weight Per Cent			Temp. (° C)	Time	Phases
	$KAlSi_2O_6$	$NaAlSi_2O_6$	$CaAl_2Si_2O_8$			
	Points in the Feldspar Field—*Continued*					
1.504....	60	20	20	1,255	48 hr.	Lots of feldspar and rare leucite in glass
				1,260	24 hr.	Lots of feldspar in glass
				1,305	24 hr.	Moderate amount feldspar in glass
				1,310	27 hr.	All glass
1.500....	62	23	15	1,265	48 hr.	Small amount feldspar in glass
				1,270	48 hr.	All glass
				1,255	72 hr.	Lots of feldspar and rare leucite in glass
				1,260	48 hr.	Lots of feldspar in glass
1.497....	10	80	10	1,240	4 days	Small amount feldspar in glass
				1,245	4 days	All glass
1.498....	20	70	10	1,235	4 days	Rare feldspar in glass
				1,240	72 hr.	All glass
1.498....	30	60	10	1,230	72 hr.	Small amount feldspar in glass
				1,235	4 days	All glass
1.498....	40	50	10	1,225	48 hr.	Rare feldspar in glass
				1,230	24 hr.	All glass
1.498....	50	40	10	1,220	72 hr.	Small amount feldspar in glass
				1,225	48 hr.	All glass
1.497....	57	33	10	1,215	24 hr.	Small amount feldspar in glass
				1,220	24 hr.	All glass
				1,185	4 days	Lots of feldspar and rare leucite in glass
				1,190	6 days	Lots of feldspar in glass
1.493....	50	47	3	1,100	4 days	Small amount feldspar in glass
				1,105	4 days	All glass
	Points in the Leucite Field					
1.504....	67	13	20	1,305	24 hr.	Small amount feldspar and small amount leucite in glass
				1,310	24 hr.	Small amount leucite in glass
				1,315	24 hr.	All glass
1.504....	70	10	20	1,335	24 hr.	Very rare leucite in glass
				1,340	24 hr.	All glass
				1,315	24 hr.	Moderate amount leucite in glass
				1,310	4 days	Moderate amount leucite and moderate amount feldspar in glass
1.500....	65	20	15	1,265	48 hr.	Small amount leucite and small amount feldspar in glass
				1,270	48 hr.	Rare leucite in glass
				1,275	48 hr.	All glass
1.498....	60	30	10	1,210	72 hr.	Moderate amount leucite and very rare feldspar in glass
				1,215	72 hr.	All glass
1.498....	70	20	10	1,295	24 hr.	Small amount leucite in glass
				1,300	6 hr.	All glass
				1,225	48 hr.	Lots of leucite and small amount feldspar in glass
				1,230	48 hr.	Lots of leucite in glass
1.496....	80	10	10	1,380	20 hr.	Small amount leucite in glass
				1,385	20 hr.	All glass
				1,245	72 hr.	Lots of leucite and very rare feldspar in glass
				1,250	48 hr.	Lots of leucite in glass
	Point on Leucite-Feldspar Boundary Surface					
1.493....	55	40	5	1,145	72 hr.	Rare leucite and rare feldspar in glass
				1,150	72 hr.	All glass

the Geophysical Laboratory furnace thermostat (Roberts, 1925)—somewhat modified from the original design but identical in principle. Temperature was measured with a thermocouple placed nearly in contact with (about 1 mm. distant from) the small charge wrapped in a tiny platinum envelope, which is suspended in that part of the furnace determined as the "hot point." In a well-insulated furnace there is a range of about 5 mm. on either side of the "hot point" where the temperature falls off about one-half of a degree. The small platinum envelope containing the charge hangs entirely within this zone.

The Pt-$Pt_{90}Rh_{10}$ thermocouple was calibrated frequently at several fixed points defined in degrees centigrade as follows:

Palladium melting point	1,549°.5
Diopside ($CaO \cdot MgO \cdot 2SiO_2$) melting point	1,391°.5
Gold melting point	1,062°.6

RESULTS OF QUENCHING EXPERIMENTS

The results of the quenching experiments are given in table 1 and are shown graphically in figure 1. Only two fields—that of feldspar (solid solutions between soda, potash, and lime feldspars) and that of leucite (pure $K_2O \cdot Al_2O_3 \cdot 4SiO_2$

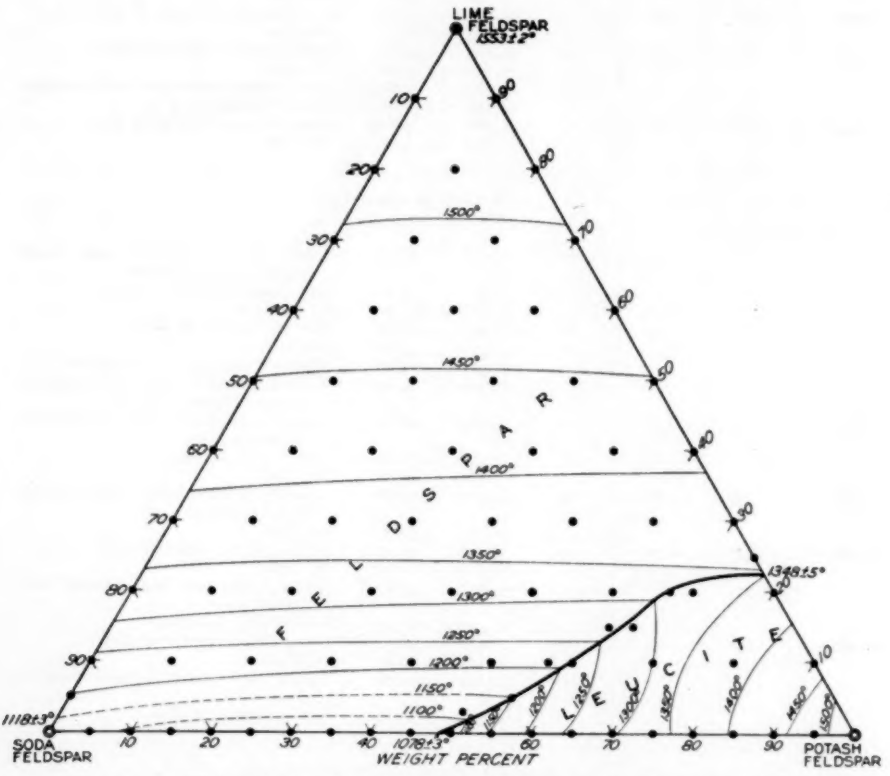

FIG. 1.—Equilibrium diagram for the system soda feldspar–potash feldspar–lime feldspar

or $KAlSi_2O_6$)—were found on the liquidus surface. No data were obtained on the composition of the feldspar crystals at or below the liquidus temperature, owing to the small size of the crystals. No adequate discussion of crystallization is possible unless the compositions of the feldspars are known.

Upon examination of quenched samples with the petrographic microscope, it was very easy to determine the temperature of the first appearance of feldspar crystals for compositions with leucite as the primary phase, because the leucite crystals lie near the glass in index of refraction and the feldspar crystals with a much higher index stand out clearly. On the other hand, it was very difficult to determine the temperature of the first appearance of leucite crystals for compositions with a feldspar liquidus, and we were able to do this for only a few compositions where the leucite appeared as large crystals which became birefringent on quenching after passing through the rapid inversion of pure leucite at about 600° (Bowen and Schairer, 1929, n. 13, p. 311).

THE JOIN SODA FELDSPAR–POTASH FELDSPAR–LIME FELDSPAR

Strictly speaking, soda feldspar–potash feldspar–lime feldspar cannot be represented adequately on a triangular diagram. Because of the incongruent melting of potash feldspar and the ap-

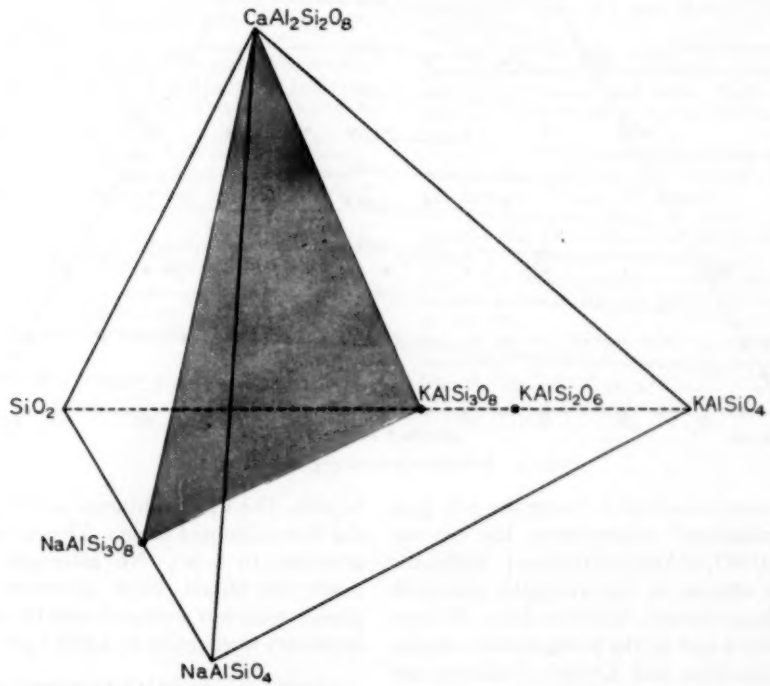

FIG. 2.—Diagram showing the position of the feldspar join (*shaded area*) in the tetrahedron $NaAlSiO_4$–$KAlSiO_4$–SiO_2–$CaAl_2Si_2O_8$, and the compositions of the feldspars and feldspathoids.

pearance of crystals of leucite (and, as a consequence, the appearance of liquids more siliceous than compositions which lie in the triangle), the results can be expressed only quantitatively in terms of a four-component system. The behavior of the ternary feldspar compositions might be represented as a triangular join in a tetrahedron[3] representing the system $NaAlSiO_4$-$KAlSiO_4$-SiO_2-lime feldspar.[4] The relation of the triangular join soda feldspar–potash feldspar–lime feldspar to silica and to the feldspathoids nepheline, leucite, and $KAlSiO_4$ (kalsilite and other polymorphs) is shown as figure 2.

[3] For a discussion of the use of a tetrahedron to represent relations in a quaternary system see Schairer (1942, pp. 246–249).

INDICES OF REFRACTION OF GLASSES

Homogeneous glasses were prepared for the forty-two ternary feldspar compositions studied, and the index of refraction of each glass was measured at 25° with the petrographic microscope by comparison with standardized immersion liquids. The values obtained are given in the first column of table 1. The values are accurate to 0.003. No attempts were made to obtain close accuracy, the glasses were not annealed, and the measurements were made in white light. The

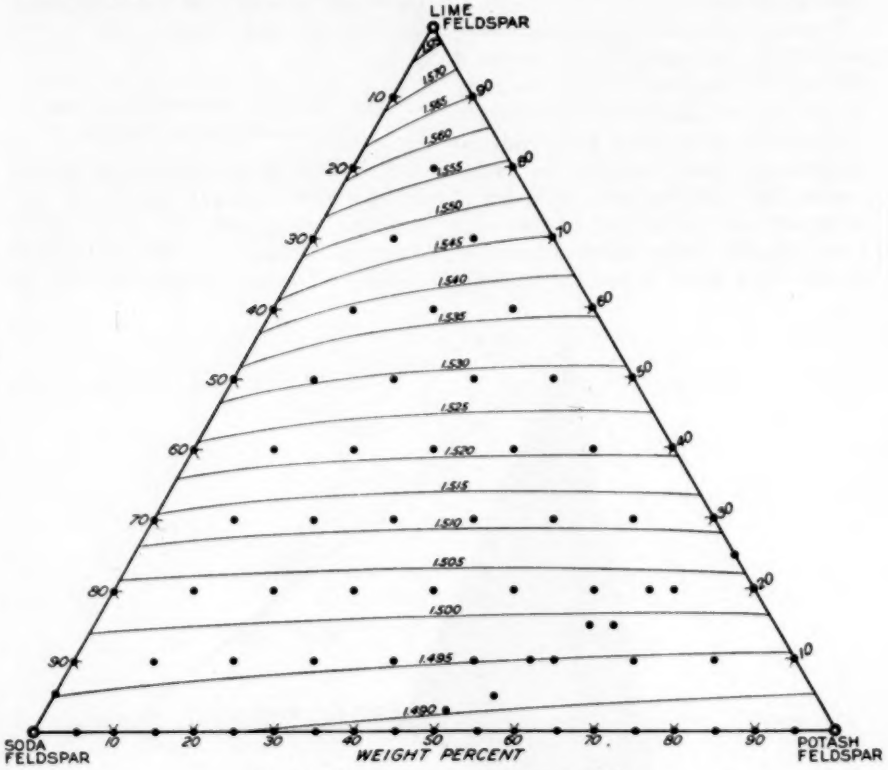

FIG. 3.—Refractive indices of glasses at 25° C.

[4] Strictly, this system is not completely quaternary because of the appearance of crystals of Al_2O_3 as a primary phase in some compositions in the system $KAlSiO_4$-lime feldspar (unpublished data of Schairer).

data are plotted in figure 3. The compositions measured are indicated by black dots, and isofracts or curves of equal index of refraction are plotted in the triangle.

VALUE OF THE DATA

The liquidus data on ternary feldspar compositions reported here is of value principally as a starting point for studies of these same compositions at lower temperatures with water as an additional component. In the presence of water at moderate pressures, even at relatively low temperatures, equilibrium between crystals and liquid is attained rapidly, and such studies should answer many questions concerning the amount of solid solution in feldspars as a function of temperature.

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DISCUSSION

ON THE BIAS OF GRAIN-SIZE MEASUREMENTS MADE IN THIN-SECTION: A DISCUSSION[1]

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INTRODUCTION

The problem of obtaining the characteristics of the size distribution of sediments from thin-section data has been attacked by Krumbein (1935), among others, who derived the equation

$$\int_0^\infty x^n Q(x)\,dx = C\int_0^\infty r^n F(r)\,dr\,,$$

where the integral on the left is the nth moment of the thin-section distribution, the integral on the right is the nth moment of the actual distribution, and C is a constant depending upon n and derived on the assumption of spherical grain shapes. He applied this theory to two sand samples—the St. Peter sandstone and a reworked glacial sand—and showed that there was good agreement between the calculated and the observed values of the first and second moments of each.

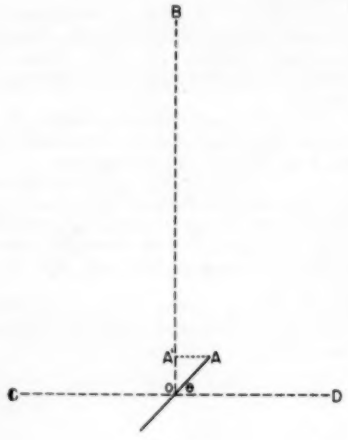

FIG. 1

In a recent paper Chayes (1950) has examined Krumbein's work and concluded that the "agreement between observed and expected values is independent of grain shape, however defined or measured" (p. 158), that this agreement "is equally uninformative about whether they [the grains] vary in size" (p. 159), and that it "tells us next to nothing about size itself" (p. 159). His final paragraph carries the implication that, since this agreement is independent of both shape and size variation of the grains, one is not justified in assuming on the basis of Krumbein's experimental results that the use of his corrections will lead to unbiased estimates of the characteristics of the original size-frequency distribution. It is the purpose of this paper to show that this implication is not justified by Chayes's presentation of the case and that his statement concerning shape, although partially true, does not follow from his attempted proof. As a matter of fact, shape affects the results in a way that neither Chayes nor Krumbein has brought out.

SHAPE AND THE BIAS CORRECTIONS

Chayes's statement concerning shape is based only on an analogy. In figure 1, a pointer of constant length, $L = 2OA$, is rotating in the BCD-plane about the origin O, with a certain angular velocity. At random intervals of time an observer looking along BO notes the length of the projection of L upon CD, and a distribution of projection lengths is obtained. This amounts to replacing the thin-section, in which a population of grains is measured at a "single instant" in time with reference to a single plane, the sectioning plane, by a single "grain," which is measured at many instants in time with reference to all possible sectioning planes normal to OB. If the angular velocity is such that the length of the projection of the pointer on OB (i.e., length $2OA'$) varies linearly with time[2] we

[1] Manuscript received April 27, 1950.

[2] Chayes erroneously states that the angular velocity of the pointer is proportional to the sine of

have an analogy to Krumbein's system because Krumbein assumed that the probabilities of cutting between equal intervals along OB are equal.[3] Therefore, we can estimate the length of the pointer by applying Krumbein's correction to the observed distribution of projection lengths.

Now, says Chayes, although it is true that the pointer describes a circle as it rotates, we can also consider the pointer as remaining at rest and the observer rotating with the same angular velocity around O, measuring the projection lengths upon a plane normal to his line of vision as before. The length L, he continues, can still be estimated, although the pointer describes no circle at all and the radius of the circle described by the observer is not given. He concludes, therefore, that shape is not a factor and that we can also measure the average c-axis of platy plagioclase crystals quenched in a lava flow or the a- and b-axes of rodlike hornblende crystals, in neither of which cases would grain shape affect the results.

This conclusion by no means follows from the analogy, for two reasons. First, Chayes is saying, in effect, that the distance of the observer is the analogue of grain shape. But it is clear that the analogue of grain shape is the path described by the end points of L, not that described by the observer. At any instant in time the length of projection of the pointer depends only upon L and the angle it makes with the projection plane; this length is entirely independent of the position of the observer along the line of sight when he makes the measurement. Moreover, it must also be clear that, with a given angular velocity, the distribution of projection lengths will, in general, be different for different rates of change in length L as the pointer rotates; that is to say, since the path of the pointer ends is analogous to grain shape, each path corresponds to a certain shape, and, in general, each will give rise to a different distribution of projection lengths.

For example, suppose the angular velocity is proportional to sec θ and L is constant; we then have the analogue of spherical grains, and the distribution function will be given by

$$P(x)\,dx = \frac{x\,dx}{L\sqrt{L^2 - x^2}},$$

as Krumbein has shown. Now, however, suppose the pointer so varies in length as it rotates that its ends describe a straight-line path. For example, if a plane is passed through any four coplanar vertices of an octahedron, the outline of the cross section so formed is a path of this sort, which might then be considered the analogue of octahedral shape. The equations of the four straight lines forming this path are:

$$\begin{aligned} x + y &= \tfrac{1}{2}L_{\max}, \text{ in the first quadrant,} \\ -x + y &= \tfrac{1}{2}L_{\max}, \text{ in the second quadrant,} \\ -x - y &= \tfrac{1}{2}L_{\max}, \text{ in the third quadrant,} \\ x - y &= \tfrac{1}{2}L_{\max}, \text{ in the fourth quadrant,} \end{aligned}$$

where $L_{\max}$ is the maximum length of the pointer. Because of symmetry, only the first quadrant need be considered, so that in this case the distribution function becomes

$$P(x)\,dx = \frac{dy}{L_{\max}} = -\frac{dx}{L_{\max}}.$$

In order to estimate the length L from the observed distribution of projection lengths, we must find the first moment, or mean, of the distribution. In the circular case we haye

$$m = \frac{1}{L}\int_0^L \frac{x^2 dx}{\sqrt{L^2 - x^2}} = \frac{\pi}{4}L;$$

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the angle it makes with CD in order for its projection on OB to vary linearly with time. If we denote this this angle by θ and the length of the projection of the pointer on OB by y, then

$$y = L \sin \theta$$

and

$$\frac{dy}{dt} = L \cos \theta \frac{d\theta}{dt} = K,$$

where K is a constant. Since angular velocity, ω, is defined as time rate of change of angle, we have

$$\omega \equiv \frac{d\theta}{dt} = \frac{K}{L \cos \theta} = \frac{K}{L} \sec \theta,$$

so that the angular velocity is proportional to sec θ, not sin θ.

[3] That is to say, the pointer projection length on OB must increase equal amounts in equal time intervals for the analogy of the rotating pointer to fulfil the requirements of Krumbein's probability assumption. Thus if, while the pointer is in the first and third quadrants, its projection on OB varies from o to $2OA'$ in the time interval Δt, then in the interval $2\Delta t$ this projection must be $4OA'$; in the interval $3\Delta t$, $6OA'$; and so on until the pointer is parallel to OB. Because of symmetry, the same reasoning applies when the pointer is in the second and fourth quadrants, except that here the projection length is diminished by $2OA'$, $4OA'$, $6OA'$, etc., in the intervals Δt, $2\Delta t$, $3\Delta t$, etc.

and in the linear case we have

$$m = \frac{1}{L_{\max}} \int_{L_{\max}}^{0} -x\,dx = \tfrac{1}{2} L_{\max}.$$

Thus, to estimate L we must multiply the observed mean by $4/\pi$ in the circular case and by 2 in the linear case. Still other bias corrections would have to be applied if we assumed the ends of L to describe a sinusoidal path or a parabolic path or any other type of path. Therefore, agreement between observed and expected results in a measurement can occur only where the correction factors are derived from equations that adequately describe the shape of the grains being measured. Such agreement could not be obtained by applying Krumbein's corrections to measurements of platy plagioclase or rodlike hornblende crystals because the shapes of these crystals are best described in terms of lines and planes, not circles.

The artificiality of rotating the observer and the projection plane about the stationary pointer in order to have the pointer ends describe no path at all proves nothing and by no means nullifies the discussion just presented because the determining factor is the path described by the pointer ends with reference to the observer and the projection plane, irrespective of which system is in motion. If we fasten the minute hand of a clock to a piece of glass and allow the works to rotate freely about the fixed hand, we will still be able to obtain the correct time, although the hand does not move at all; the face will merely rotate about the fixed hand, and the apparent motion of the latter relative to the former will provide us with the necessary information. If we wish to measure the rate of lengthening of the shadow of a tree, we will obtain the same results whether we consider the sun as moving about a fixed earth or the earth as rotating in the light beam cast by a fixed sun. It is entirely immaterial which of the two systems, "observer-projection plane" and "pointer," is rotating and which is fixed or even whether both are rotating in either the same or opposite directions. The important fact is that the path described by the pointer ends relative to the observer and projection plane is precisely analogous to grain shape. It has already been demonstrated that variations in this path cause variations in the distribution of projection lengths. Therefore, by analogy, variations in grain shape will result in varying size distributions measured in thin-section, so that agreement between expected and observed results of such measurements will depend upon the selection of proper correction factors, namely, those based on equations that adequately describe the shape of the grains.

The second flaw in reasoning from this analogy has already been hinted at, namely, that when we measure projections of the pointer upon a plane we are, in effect, measuring the diameters of circular sections of a sphere produced by all possible sectioning planes parallel to the plane of projection, whereas in the case of a thin-section we are measuring circular sections of many grains produced by only one of these sectioning planes. It is apparent, therefore, that rotating pointer measurements correspond to thin-section measurements only in those cases in which the shapes of the grains are such that the dimension measured in thin-section has the same length as the projection of the pointer upon the plane of the thin-section (i.e., the plane normal to the OB-axis) and in no other case. This condition is fulfilled only by the sphere generally and by special cases of other shapes.

To illustrate this point, let us consider, first, the case of spherical grains. Here the analogue is a pointer of constant length L. When the pointer makes an angle θ with the plane of projection, its projection length is exactly equal to the diameter of the circular section of the sphere produced by a sectioning plane that is at a distance $L \sin \theta$ from the sphere center. This situation holds for all angles, and the analogy, therefore, holds also. Now, however, consider a different shape—a cylinder of length l and diameter d. The projections of l and d will be equal to $l \cos \theta$ and $d \sin \theta$, respectively, for all angles, so that rotating pointer measurements will yield a distribution of lengths varying from 0 to l if we measure l or from 0 to d if we measure d. In the thin-section, on the other hand, we will usually observe elliptical sections with minor axis of length d and major axis of length $d/\sin \theta$ for most angles. Thus in every elliptical section the short dimension will be true d, and we have to make no bias correction whatsoever. The distribution of long dimensions, however, will have to be corrected by some factor other than that used in the case of projection measurements in order to obtain true l.[4]

[4] The foregoing discussion neglects (1) the situation at the ends of the cylinder, (2) the case for

The distributions in the two cases are thus markedly different, the difference arising from the fact that, in general, projection measurements are entirely unrelated to measurements made on a single section precisely because of the effects of shape. Therefore, because Chayes's analogy holds generally only for one special shape—the sphere—and breaks down for all others, we cannot with confidence draw any conclusions from it as to the effect of grain shape on the agreement between expected and observed results of thin-section measurements.

The manner in which grain shape affects the bias corrections has been brought out in Wicksell's work on this problem (1925, 1926). Wicksell applied a rigorous mathematical treatment to both spherical and ellipsoidal grain populations, taking into account not only the distances of the grain centers from the sectioning plane but the probability of a grain's being cut by this plane as well. Krumbein's solutions are thus a special case of Wicksell's, where, in the latter's notation,

$$F(r) \equiv f(r),$$

where $F(r)$ denotes the size-frequency distribution of the actual population and $f(r)$ the size-frequency distribution of those grains cut by the sectioning plane. It is shown that in this simplified case the same corrections apply, whether the grains are spheres or ellipsoids of any eccentricity and of any orientation. This result comes about because of a special relation between the sphere and the ellipsoid, namely, that the ratio of the area of any section of an ellipsoid to the area of a parallel central section is the same as the ratio of the area of a similarly situated section of a sphere to the area of its parallel central section. This is not too surprising, in view of the fact that mathematically the sphere is a special case of the ellipsoid; the circle is often considered an ellipse with zero eccentricity.

In Wicksell's general solution for the ellipsoidal case the departure from sphericity results in different correction factors; however, he has calculated these factors for the special cases of the prolate and oblate spheroids, and it turns out that, even with eccentricities as high as 0.7, the values for the first five moments do not differ much from those for the spherical case. Thus, whether one applies Wicksell's general solutions or Krumbein's simplified ones, there is little error involved in assuming the grains to be spherical.

It should be observed, however, that both Wicksell and Krumbein obtained their solutions by using the equation of the ellipse; Krumbein, in fact, used the special case of an ellipse with zero eccentricity, namely, a circle. Thus Chayes's statement concerning the effect of shape is true, but only with regard to grains that can be assumed to have ellipsoidal outlines. As has already been shown, if we assume grains with octahedral shape, the distribution of "diameters" measured in thin-section will differ from that of the spherical case and the bias corrections in this case will be different from those of either Wicksell or Krumbein. Now, since an orthorhombic dipyramid bears the same relation to an octahedron that an ellipsoid bears to a sphere with respect to the property mentioned above (i.e., the ratio of areas of similarly situated parallel sections), the correction factors derived for the octahedron will apply equally well to a tetragonal or orthorhombic dipyramid. The same reasoning can be applied to other three-dimensional shapes: thus the corrections for the sphere will apply to the ellipsoid; those for the octahedron, while different from those for the sphere, will apply to the orthorhombic dipyramid; and a still different set derived for cubes will work also for all rectangular parallelepipeds. An important fact concerning the effect of shape on the bias cor-

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$\theta = 90°$, and (3) the case for $\theta = 0$ and small angles in the vicinity of $\theta = 0$. Under the first head, there is the complication that cuts near the ends will produce section boundaries that are part ellipse and part straight line. In the second case, all sections will be circles of diameter d, and thus no bias corrections are required for this particular cylinder orientation. In the third case, cuts through the ends will produce rectangular sections of length l for $\theta = 0$ and of length slightly greater than l for small angles near $\theta = 0$, the limiting values of such angles depending upon the relative dimensions of the cylinder. For the special case $\theta = 0$ no bias corrections are required to obtain true l, whereas d values—i.e., the short dimensions of the rectangular sections—must be corrected by Krumbein's correction factors to obtain true d. However, if l is sufficiently large compared to d, these less usual sections will be in the minority, the majority consisting of the elliptical sections discussed in the text. In any case, these complicating situations do not destroy the illustrative value of the cylinder because it was selected only to show that, in general, the effect of particle shape is to produce thin-section size distributions that are completely unrelated to pointer-projection size distributions.

rections consequently emerges, namely, that the corrections are independent of shape defined on the basis of sphericity alone but *do* depend upon shape defined on the basis of the equations describing the grain surfaces.

It may be pointed out that here we have a case in which the mathematical model (i.e., the derivation of the bias corrections) must take account not only of the experimental procedure but of the material upon which the experiment operates. As Chayes indicates, agreement between the expected and the observed results of a measurement can be expected where the experimental procedure closely corresponds to the mathematical model we construct. However, it is important to note that we must first examine the material in order to set up the proper mathematical model. Thus if we wish to measure Chayes's platy plagioclase crystals or rodlike hornblende crystals, we must first examine the material and select the proper equations describing the grain shapes in order to construct the proper mathematical model. Otherwise, agreement of results cannot be expected, for application of ellipsoidally derived corrections will result in errors that may be great or small, depending upon the amount of error involved in ascribing ellipsoidal outlines to shapes defined by planes and straight lines.

SIZE VARIATION AND THE BIAS CORRECTIONS

In discussing the effect of size variation on the bias corrections, Chayes has satisfactorily demonstrated that the values of these corrections are independent of whether or not the grains vary in size. This is indeed a fortunate—in fact, necessary—situation because, if the values did depend upon the size variation of the original grain population, we would be faced with the seemingly insoluble problem of obtaining information by applying corrections the values of which themselves depend upon the unknown information we are using them to obtain. However, in his discussion and in the implications that follow he seems to have overlooked an important point, namely, the existence of a distribution of thin-section "diameters"—Krumbein's $Q(x)$ curve.

There are, after all, three elements to be considered in a measurement of this sort. The first is the size distribution of an actual grain population; this is the unknown information we are seeking. The second is an operation acting on this population so as to produce a bias, the correction for which we derive mathematically on the basis of certain assumptions. The third and equally important element is an observed size distribution of grain sections measured in thin-section. Now it is this third element, not the second, that tells us how the grains vary in size, for the $Q(x)$ curve bears a definite relation to the original $F(r)$ curve.

If we consider any hypothetical distribution of grain sizes and assume that all the grains in any class interval are of a size equal to the midpoint of that class interval (as is done in calculating the moments of a distribution), then for each of these classes there will be a distribution of thin-section sizes that has the form of Krumbein's $P(x)$ curve. The $Q(x)$ curve will then be the sum of all these $P(x)$ curves, and it is easily verified, by drawing a number of distributions, that to each $F(r)$ curve there corresponds a unique $Q(x)$ curve. Inasmuch as corresponding moments of the two curves have a constant ratio, a change in the moments of one will result in a corresponding change in the moments of the other. Thus, although the *value* of this ratio is independent of size variation, the $Q(x)$ curve is not, and we can therefore obtain a reasonable estimate of the desired moments from the observed moments.

The values of the constant ratios involved depend upon the assumptions of randomness and upon the equations assumed to describe the grain shapes; that is, both the experimental procedure and the material dealt with must be considered in deriving the bias corrections. Krumbein's experiments show that his assumptions sufficiently approximate the physical facts that his corrections can be used to give reliable results in measurements of similar sediments.

SIZE DEFINITION IN THIN-SECTION MEASUREMENT

The definition of size is a problem separate from that of correcting the bias of a thin-section measurement. The bias corrections, it is true, are not wholly independent of the size definition. For example, Wicksell found it convenient, in developing his corrections for ellipsoidal grains, to define size as the geometric mean of the long, intermediate, and short diameters, so that his corrections can be applied only to distributions where the size is so measured. With Krumbein's simplified theory, however, the corrections can be used whether we measure the long diameters, the short diameters, or their arithmetic or geometric means. The point is that, assuming

size to be defined on the basis of a certain parameter, the sectioning process displaces the moments of the size distribution of this parameter by a fixed amount to give the moments of the thin-section distribution of this same parameter.

Now all we demand of a thin-section measurement is that, given a certain size definition, we have a reliable set of correction-factor values that will enable us to estimate from the observed distribution the distribution we would obtain from a sample of loose grains. In other words, if we measure long diameters (or short diameters or any mean of the diameters) in thin-section, we would like our correction factors to give us a reliable estimate of the distribution of long diameters (or short diameters or any mean of the diameters) we would measure in the sample of loose grains. This is all we demand, and this is all we get. We can bridge the gap between the thin-section and the loose grains, but, once having obtained the loose-grain distribution, we still face the problem of what is size itself the same as if we had started with a sample of loose grains in the beginning. The problem is thus outside the realm of the bias-correction problem, and it is unfair to ask that the latter solve the former, since the definition of size is a problem that is inherent in any and every kind of size analysis we make.

For example, if we make a size analysis of a glacial till, we use three different size definitions. The coarse fraction can be measured with calipers, so that "size" may be the lengths of the long diameters of the cobbles; the arithmetic or geometric means of the long, intermediate, and short diameters; or the diameters of spheres of the same volume or some other such measure. The pebble and sand fraction is usually analyzed by means of sieves, and here "size" is defined by the dimensions of the sieve openings. The silt and clay fractions are frequently analyzed by sedimentation techniques, where "size" is defined in terms of the settling velocities of quartz spheres. The distributions of these three fractions are thus not strictly comparable; but, if we have a number of till analyses, we can obtain important information by comparing the respective coarse fractions, the respective sand fractions, and the respective fine fractions.

Similarly, if we measure long-diameter distributions in thin-section and correct them to obtain the long-diameter distributions we would measure in loose-grain samples, we can obtain valuable information by comparing these results not only with other thin-section measurements but also with measurements of long-diameter distributions of loose-grain samples. Thin-section size analyses may suffer from the problem of size definition, but no more so than any other kind of size analysis.

CONCLUSIONS

In summary, then, we can list the following conclusions:

1. Because the bias corrections are derived on the basis of certain assumed grain shapes, to obtain reliable results it is necessary to select the mathematical curves that most nearly approximate the shapes involved. The values of these derived corrections thus depend upon the material being measured. In the case of sandstones Krumbein's corrections based on ellipsoidal grain outlines give reliable practical results, as his experiments have shown.

2. Chayes's statement that Krumbein's observed agreement of experimental and expected results is uninformative about the variation in size of the grains is true, but misses the point. The experiment does not attempt to tell *whether* the grains vary in size; the information concerning the size variation of the actual grain population is obtained from the distribution of thin-section diameters, not from the bias corrections. The experiment was an attempt to establish the hypothesis that, in sectioning the grains, the moments of the original distribution are displaced by a constant amount, no matter how the grains vary in size. The fact that the observed ratios of the corresponding moments of the thin-section and loose-grain distributions are in close agreement with the mathematically expected ratios indicates that this hypothesis is valid. If the hypothesis is valid, we then have reliable values for these ratios—which are the bias corrections—and, by applying them to observed thin-section distributions of similar sediments, we can obtain reliable estimates of the distributions we would observe if we measured samples of loose grains. Thus we are not eliminating the bias by knowledge about the grains (except to the extent that we derive the theoretical correction values by making certain assumptions as to the grain shapes), but we are obtaining knowledge about the grains—i.e., the moments of their size distribution—by eliminating the bias. We eliminate the bias by obtaining reliable values for the constant ratios, and Krumbein's experiments show that his values

are sufficiently reliable to give good practical results.

3. The problem of how to define size itself crops up in connection with thin-section size analyses just as it does with every other type of size analysis, but the fact that it exists no more destroys the value of this method than it does that of any other. At the present stage of the science of sedimentology every technique of size analysis requires the setting-up of an arbitrary definition of size to fit the procedure used and the computations based on such procedure. The thin-section method is no more guilty in this respect than is any other method. Although it is certainly desirable to have a more uniform and fundamental definition of size, until the day arrives when such a definition appears we can obtain much useful information for comparative and other purposes from the thin-section method as it stands at present.

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ON THE BIAS OF GRAIN-SIZE MEASUREMENTS MADE IN THIN SECTION: A REPLY[1]

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Mr. Greenman seems to overlook the fact that my paper was intended solely as a discussion of certain aspects of Dr. Krumbein's, and where he charges me with errors of interpretation, it is usually because he has not taken the trouble to compare our work with sufficient care. Granting that lead shot do not depart sufficiently from sphericity to affect the result, Krumbein's first experiment has significance only as a test of whether or not the experimental situation justifies his choice of an "equi-distance" rather than an "equi-angle" (or some other) definition of randomness. Although he nowhere intimates that choice is involved—i.e., that other solutions are possible—it seemed to me that, except for this possibility, he might have rested with either the mathematics or the experiment and had no need of both.

Carried away by this discovery, I was guilty —although not quite so guilty as Mr. Greenman suggests—of understating the difficulty introduced by shape differences. Krumbein did seem to regard the agreement between experiment and arithmetic as in some fashion a verification of the sphericity of the grains in his sandstone specimens, for he went so far as to discuss the results in terms of roundness and sphericity indices. But if, with the same assumptions, shapes other than spheres or circles give the same agreement, the agreement is not *ipso facto* indicative of the sphericity of the grains. It was in this sense that I suggested the two were "independent," and I agree with Mr. Greenman that the word is too strong. Krumbein's Minnitaki sand grains do not look particularly round to me, yet evidently they are "sufficiently round." It seems quite possible that many acicular crystals might also be "sufficiently linear." At any rate, the core of my argument was not that one *should* measure acicular crystals in this fashion but rather that one *should not* regard agreement between observed and expected values as necessarily related to roundness or sphericity.

Mr. Greenman is correct about my erroneous

[1] Manuscript received February 10, 1951.

specification of angular velocity for the equal-distance criterion in figure 1, but his analogical discussion of the pointer analogy is misleading. There is hardly any need for a model of the sphere other than the sphere itself. If he chooses, he may regard either the path of the pointer or the "distance of the observer"—whatever this may mean—as analogous to grain shape. But what about the pointer itself? It is intended as a model of a crystal very much longer than it is broad or thick, and its length may be taken as either the maximum or the minimum dimension of such a crystal. Its path is intended as a model of rotational movement in metamorphism or, less adequately, of movement in turbulent flow.

I should like to assure readers of the *Journal* that I did not overlook "the existence of a distribution of thin-section 'diameters,'" for it was the existence of several such that first drew my attention to the problem. The question was whether one should apply the correction factors before comparing results. Where there is some possibility of an independent check, as in unconsolidated sands or lead shot, there may be some excuse for multiplying observed values by constants. Where there is no possibility of such a check, as in the "grain-size" measurements made on most metamorphic and igneous rocks, this is just playing with numbers. The observed distribution of x contains just as much information as that of kx, and, in dealing with it, one is much less likely to overestimate the importance of small differences.

Finally, this is a good opportunity to correct two errors Mr. Greenman has overlooked. In my note (p. 157) it is asserted that from the measurement of random chords the proper estimate of d would be $2\bar{m}$. This was based on a guess that the correction for distances of centers from surface would be of the equal-distance type suggested by Krumbein, while that for displacements from the center in the plane of the section would be of the equal-angle type. From experimental work on briquetted lead shot it now appears that this was a bad guess. Both corrections are of the Krumbein type, and $d = (16/\pi^2)\,\bar{m}$ gives a much better approximation than $d = 2\bar{m}$.

In a discussion of the insensitivity of the correction factors to variation in grain size (p. 159), it is alleged that the observed second moment is to be multiplied by $\frac{3}{2}$, "whether d be regarded as fixed or as variable," but it is obvious that, if d is fixed, its second and higher moments will be zero. Now the second moment of measurements made in section will certainly be greater than zero, and it follows that from data obtained in this fashion we will never determine that the grains are all of the same size; in fact, it will always appear that they differ, though, of course, the variation may be greater in one batch than in another. Mr. Greenman overstates the case when he agrees with something I did not say, viz., that agreement of observed and expected results is "uninformative about the variation in size of the grains." The thing we cannot find out is *whether there is* variation in size. If it exists, we shall get some measure of it, but from measurements made in section it will always appear to exist.

Mr. Greenman's decision that this matter is inconsequential again reflects his failure to compare my work with Krumbein's. I guessed that Krumbein "used a number of lead shot, all of the same radius," because he suspected that variation in grain size would upset the check between theory and experiment. The guess may be a poor one, but, unless Mr. Greenman has some better explanation of this choice of material, it is he, not I, who missed the point.

Mr. Greenman is certainly correct in concluding that "shape affects the results in a way neither Chayes nor Krumbein has brought out," and if he too has failed to bring it out in simple usable form, no one familiar with the problem will wish to cast the first stone. His work shows promise, but it might have shown more than that if he had not spent so much time treating a highly specific argument as if it were a statement of general principles.

REVIEWS

Le Modelé périglaciaire. By J. TRICART. Paris: Centre de Documentation Universitaire, 1950. Pp. v+267; figs. 84. Planographed.

Le Modelé périglaciaire is an outline prepared for students as part of a more extensive course in geomorphology. Two volumes dealing with structural geomorphology—(1) *Le Relief des côtes* and (2) *Les Types de bordures de massifs anciens*—have already appeared. This volume is the first of a proposed series dealing with climatic geomorphology. In its entirety this proposed course reveals courage and vision on the part of its authors, A. Cailleux and J. Tricart. It represents an approach to the study of geomorphology which, though apparent in the attitudes of many geomorphologists in both America and Europe, has heretofore not been presented at the student level. It breaks sharply away from the trends of American textbooks and offers, instead, an approach which is more likely to render the subject applicable to related fields of study. It implies that geomorphology can no longer be studied at the broad sweep of the arm if, indeed, this were ever possible. This system of geomorphology, as described by Tricart, consists of three parts:

Part

1. Azonal processes for the sculpture of relief
   - Pluvial erosion
   - Littoral erosion
   - Eolian erosion
2. Climatic geomorphology
   - Topography of the cold lands
     - Periglacial topography
     - Glacial topography
   - The topography of arid regions
   - The topography of humid, forested lands
3. Structural geomorphology
   - The classification of rocks and the mechanism of their attack by erosion
   - The structural types of relief
     - Relief of hills
     - Relief of depressions
     - Relief of folds
     - Relief of volcanoes
   - The major structural units
     - The geomorphology of the old massifs
     - The sedimentary basins
     - The recent mountain chains.

It is still too early to see how this outline will be treated in volumes proposed but not yet written. Broadly defined, these units could encompass the subject. Perhaps, however, a fourth part should be included, such as:

4. Historical geomorphology
   - The effect of tectonic changes
   - The effect of climatic changes
   - Unconformities.

This approach, particularly as it is exemplified in *Le Modelé périglaciaire*, shows how detailed is the knowledge of climatology and how minute are the observations of soils required in modern studies of geomorphology. They are fully as important as structural geology and petrology. All these topics have become equally essential tools in current geomorphic thinking. Geomorphic studies described here show how very local climatic conditions may play an important part in weathering and erosion. Microclimatic studies, particularly when related to the climate of the soil, must therefore become increasingly important.

The discussion of periglacial topography is presented here in a simple, clear, and direct style. The outline form which Tricart has used makes for easy reading and easy reference. It has enabled him to condense a remarkable amount of information into a relatively small volume. In some instances, however, there are statements or points of argument which merit amplification and further discussion. Periglacial topography is thoroughly considered in three chapters. In the first chapter the periglacial domain, the climatic environment of periglacial regions, and the behavior of ice in the soil are treated. The second chapter is devoted to cryopedology, or periglacial soils. Here Tricart, in order to avoid commitment to a genetic hypothesis, has introduced a descriptive classification of periglacial soils. This classification is generally applicable and is therefore given below in translation:

CLASSIFICATION OF PERIGLACIAL SOILS

A. Soils with geometric form
   1. Soils with closed form
      - *a*) Simple forms with arrangement or sorting (normal polygons and stone rings)
      - *b*) Simple forms without sorting (tundra polygons)
      - *c*) Complex types (subdivided polygons)

2. Soils with open form
   a) Striped soils
   b) Cryoturbation folds (i.e., solifluction terraces)

B. Amorphous soils
   1. Soils with protuberances
      a) Turf-covered mounds
      (Thufur, earth mounds, *Erdhügelböden*)
      b) Tundra hillocks
      (ground-ice mounds, pingos, palsen, hydrolaccoliths)
      c) Twisted peat moss
      (*Streifensümpfe, Strangmoore*)
      1*a*) Sloping soils with folds
         2*a*) Soils with steps
         (sheep tracks, terracettes)
         2*b*) Soils with swelling by cryoturbation
      1*b*) Level soils without sorting
         2*a*) Mud fields
         2*b*) Stone fields
         2*c*) Stone pavement
      1*c*) Block streams and sorted rubble
         2*a*) Sorted rubble
         2*b*) Rubble flows

In the course of the discussion a new word, "cryosol," is introduced. This word is a synonym of "congeliturbate" and of "frost soil." It applies to any body of material modified by frost-action, both that which is orderly and that which has an amorphous arrangement. As a general term it has merit, but it is formed of an odd combination of a Greek and a Latin root. The third chapter deals with periglacial topography. Like the preceding, this chapter is complete in its content and logical in organization. Not only does it contain a discussion of such well-known topics as congeliturbation and rubble-mantled slopes, wind action and loess deposition, and such features of poorly drained conditions as the thaw-lakes and tundra hillocks, but it also includes a discussion of much less thoroughly developed concepts, such as differential congelifraction and the asymmetry of valleys. The argument would, however, have been strengthened if, while discussing the forms produced by frost, more attention had been given to criteria for distinguishing similar nonperiglacial forms produced under the combined action of chemical decomposition, gravity, and viscous soil flowage. At the end of each chapter there is an excellent annotated bibliography of important papers and books related to periglacial geology. In all, 262 references are cited.

Two concepts included in this discussion are noteworthy for their novelty. Certainly they merit further discussion. One, attributed to a personal communication from Cailleux, is the concept that chemical weathering can, by forming an impervious residuum, render a surface immune against the action of frost. The second is the concept that peneplanation is impossible in at least some periglacial environments. Tricart assumes the dominant periglacial process of denudation to be equiplanation, as described by Cairnes from Alaska in 1912, whereby a gentle surface is formed, by the destruction of the slopes and filling of the valleys, at an altitude intermediate between that of the summits and the valley bottoms. It seems to the reviewer that this is a late mature or early old age stage of the periglacial cycle of erosion. It may represent the maximum development of this cycle, where periglacial processes have operated only within the relatively short span of Pleistocene time. Congelifraction and congeliturbation do not, however, cease at this stage. Wave action and congelifraction produced by sea water must continue to occur along the coast. Nick-points will therefore continue to be formed at the shore line and must retreat headward along the flowing streams. Base level is thus as significant in periglacial regions as in temperate regions. A frost-produced peneplain, or surface of cryoplanation, must therefore remain a possibility, as was pointed out by Bryan in 1946.

This excellent outline provides us, then, with the most complete summary available of periglacial geology and with an approach to the subject which is both logical and effective. It is valuable to the student of soils and surface geology for its content and to the teacher for its educational approach.

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Historical Geology. By CARL O. DUNBAR. New York: John Wiley & Sons, 1949. Pp. xii+567; figs. 380. $5.00.

Seventeen years ago, when reviewing the third edition (1933) of "Schuchert and Dunbar," Dr. Carey G. Croneis, president of Beloit College, made the following statement (*Jour. Geology*, vol. 42, p. 335):

> From the sidelines it has been interesting to watch the metamorphosis of that fine old Yale textbook team of Pirsson and Schuchert. On the physical side Longwell, Knopf, and Flint have now been in the line-up in place of Pirsson for some time; and recently a new man, Carl Dunbar, has been added

to the historical roster . . . it is safe to say that the new men can play the game quite as well as their elders. Of course, one does not have to read far . . . to see that the veteran Schuchert is still captain, yet nevertheless there is readily discernible a new attack. . . .

In a reply to this review by Dr. Croneis, a little poem was soon forthcoming from a "down-easterner," and I think it interesting and worth repeating here:

I'm happy to see, says hoary C. C. [Carey Croneis]
The veteran sports reviewer
That the new Yale bunch still carries the punch
That makes their competitors bluer.

I've watched Yale play since that far-off day
When I put on my first long trouser,
They're a fine lot of men, I'll encourage them, then,
I'll give them a good write-up, yowser.

They face somewhat east and their planets at least
From the only true gospel have run far
But I'll praise without stint,
Knopf, Longwell, and Flint,
Cap. Schuchert and even young Dunbar.

These interesting little side lights on earlier reviews of the fine textbooks that have consistently come from the "Yale team" give a humorous, yet accurate, picture of the historical development of this series. Another chapter must be written on the development of the "Yale team," for now Dunbar is the coach-captain-player of the historical team and has the sole responsibility for every play. If Dr. Croneis should review another "game," I am sure he would say that the style of play is the same, but the attack is more polished and the running game much smoother. The "Yale system" is so thoroughly entrenched that even Dunbar continues to use "we."

This new *Historical Geology* is divided into seven principal parts, namely, (1) "Prologue"; (2) "Before the Cambrian"; (3) "The Paleozoic World"; (4) "The Mesozoic World"; (5) "The Modern World Unfolds"; (5) "Epilogue"; and (7) "An Introduction to Animals and Plants." There are 567 pages in all, with some 450 exceptionally fine pictures and illustrations, many completely new, making up about 33 per cent of the book. Pictures repeated from the fourth edition were considerably improved by increasing the size and reproducing them on a better grade of paper. By placing most illustrations at the top or bottom and extending them to the edge of the page, the space allotted pictures was more than doubled at little or no expense to the space for text. Many of the pictures are original photographs by the author; these are supplemented by some excellent air photos, photographs of outstanding paintings and murals of various museums, as well as many taken by professional photographers. The book is printed with the aid of large and clear type on superior paper, both of which make it a better textbook from the student's point of view. The sentences are more concise, yet convey the same meaning as the longer, more involved sentences of the fourth edition. The phraseology is modern.

Sixty-seven pages are given to the four chapters of the "Prologue." The titles of these chapters are (1) "Records in Stone," (2) "The Scale of Time," (3) "Fossils, a Living Record of the Dead," and (4) "The Constant Change of Living Things." The basic material varies but little from the fourth edition, but it has been rearranged to advantage.

The second part, "Before the Cambrian," is divided between (1) "Origin of the Earth" and (2) "The Cryptozoic Eon: Pre-Cambrian History of the Earth." The words of the author express the objectives:

> To avoid the monotony inherent in the systematic account of period after period, emphasis is varied from chapter to chapter. In discussing the Cambrian, for example, considerable space is devoted to paleogeography and to the bases for subdividing the rocks into series and formations; in the Ordovician chapter, the Taconian orogeny is discussed at length because it illustrates the principles used in recognizing and dating all later orogenic disturbances. In this way, general principles of interpretation are developed one after another in the early chapters, leaving room in later ones for greater detail. This we believe to be fitting since human interest in rocks and fossils increases as we approach the modern world.

To the reviewer it is difficult to reconcile the author's statement, "general principles of interpretation are developed one after the other in the early chapters," with the fact that only 35 pages, or the same number apportioned the Cretaceous, is assigned to the entire pre-Cambrian. Some geologists believe that there are enough principles involved in a study of the pre-Cambrian to allow this section more space, for chronologically it is here we must first study. It is a matter of question when one thinks "human interest in rocks and fossils increases as we approach the modern world." Maybe the teachers could create an interest in the pre-Cambrian if more emphasis were placed on this study and the principles involved. The author states that "about three-quarters of all geologic

time had transpired before the Cambrian period"; yet only 6 per cent of the book—35 out of 567 pages—is devoted to a discussion of the history of 75 per cent of all geologic time. It appears that we continue to sell the pre-Cambrian "short."

Part 3, "The Paleozoic World," is discussed in the next 179 pages. Dunbar has done an excellent job of reorganizing the chapters, simplifying sentences, and reducing the material in the paragraphs without loss of value and meaning. The paleogeographic panels are from Schuchert's latest unpublished maps. The maps of the panels are nearly three times the size of those in the fourth edition, but they are arranged in reverse order, that is, the map showing the paleogeography of the earliest or older part of the period is placed at the top of the page, and the youngest is at the bottom. These maps are framed with clouds and in the words of the author, "the clouds have no meteorologic significance; they are merely a device to hide critical areas for which evidence is lacking or inconclusive." This type of panel is used throughout the Paleozoic and Mesozoic, but the simpler, minus-the-clouds type is used for the Cenozoic, and I believe, aside from the aesthetic aspect, that it is much better for the student.

Correlation tables were omitted in the first printing, since the author thought them too technical for the student and not complete enough for the teacher. Footnote references to the correlation charts of the Committee on Stratigraphy of the National Research Council are found in appropriate places. At the request of professors the correlation charts are now included in every copy. This should widen the scope of the book.

Most of the plates of fossils are the same as those used in the fourth edition, but, by reproducing them on the better paper, the minor characteristics of the fossils are more easily deciphered. The enlarged photographs add evidence to the material of the text. Many of the old pictures and drawings depicting Paleozoic life are replaced by enlarged photographs of a great mural (by Rudolph Zallinger) in the Yale Peabody Museum.

The story of the Mesozoic world is told in the next 85 pages. The material is essentially that of the previous edition. A change was made in the section showing the contact of the Triassic and Jurassic. Attention is called to the finding of a small bipedal dinosaur in the Navajo sandstone of Arizona, and tracks of dinosaurs have been discovered in the slabby sandstone between the Wingate and Navajo—this may make the Wingate sandstone late Triassic and not early Jurassic, as indicated in the fourth edition. New pictures, aerial views, cross sections, and block diagrams add much to this section of the book.

The four chapters of part 5, "The Modern World Unfolds," are (1) "Physical History of the Cenozoic," (2) "Ice Sculptures the Final Scene," (3) "Mammals Inherit the Earth," and (4) "The Coming of Man." The physical history of the Cenozoic is written as a unit rather than being broken into individual periods. To some extent the chapter on ice sculptures has been rewritten. The Keewatin and Labrador centers of ice accumulation have been combined into one now called the "Laurentide Ice Center." Recent studies and implications of pollen analyses of interglacial deposits with reference to interglacial climates and history are discussed. A part is assigned the importance of fossil mammals as a guide to the history and climate of the Pleistocene. A more complete calendar of Pleistocene time (by Flint) replaces the calendar of the previous edition. It is interesting to note that Dunbar omitted the controversial subject, "causes of glaciation"; this topic was given several pages in earlier editions.

"Man's family tree" is covered more completely with the anticipation of the chapter to follow, the "Coming of Man." This is generally a very interesting part to the beginning student in geology, and often it is hurriedly passed over by some authors. In the "Coming of Man" a more complete calendar of human history (by Flint) has replaced the old. Recent discoveries, including those in Mexico, are discussed.

The Appendix, "An Introduction to Animals and Plants," was prepared for the beginning student in geology without previous training in biology. The chapter begins with a simple classification of animal and plant kingdoms, following which the major groups of animals and plants are reviewed. Most teachers will find this a very important part of the book, and the students lacking a background of biology should find it extremely helpful.

In summary, the simple and lucid writing; the carefully selected, beautifully reproduced photographs and illustrations; the well-arranged and clearly set-off paragraphs; the simple but adequate diagrams; the treatment and organization of the up-to-date material; the rather complete list of references and collateral read-

ings; and the general excellence should make this work by Dunbar an outstanding textbook for the teacher and student of historical geology.

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Pleistocene Terraces of the Susquehanna River, Pennsylvania. By LOUIS C. PELTIER. (*Pennsylvania Geological Survey Bull.*, no. G 23, 4th ser.) 1949. Pp. 158; figs. 49; tables 46.

Concealed by this modest title is the first comprehensive American study of terraces from the point of view of Pleistocene climatic fluctuations in both glacial and periglacial areas. Recent European concepts of periglacial weathering and mass movements are supported by field evidence and are integrated with terrace interpretations and Pleistocene chronology in general. From it emerges the view that more landscape features than have been generally realized are inherited from Pleistocene conditions and that climatic cycles provide a natural basis for correlating Pleistocene events.

The subject matter can be divided into six general parts: (1) introduction; (2) summary of glacial geology; (3) weathering and soils; (4) periglacial deposits; (5) description of terraces; and (6) Pleistocene climate of central Pennsylvania.

Pleistocene deposits in the area include pre-Illinoian gravels, Illinoian till and gravels, Binghamton (Tazewell) till and gravels, Valley Heads (Cary) gravels, and Mankato gravels. The gravels represent both glacial outwash and periglacial deposits resulting from intensified frost-action and mass movements. Only the Mankato gravels are exclusively periglacial. Sequences of (1) marginal and frontal kame terraces, (2) valley-train terraces, and (3) periglacial terraces are recognized in going away from the glacial boundaries. Deposits of various ages are differentiated on the basis of degree of weathering and soil development, lithology, stratigraphic position, and physiographic relations. Correlations with events outside the region are based on the principle that the climatic cycles of the Pleistocene were contemporaneous and of world-wide extent.

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Structural Geology of Canadian Ore Deposits: A Symposium. Arranged by a COMMITTEE OF THE GEOLOGY DIVISION, CANADIAN INSTITUTE OF MINING AND METALLURGY. Montreal: Mercury Press, Ltd., 1948. Pp. 948.

This volume is divided into four sections. Chapter i, "General Papers," contains "The Cordilleran Region of Western Canada," "The Canadian Precambrian Shield," "Mineral Occurrences in the Appalachian Region of Canada," and "Relationships of Minor Structures to Gold Deposition in Canada."

Chapter ii treats of the Cordilleran region and has twenty-eight papers on the mines and mining districts of the coastal belt, interior belt, and Rocky Mountain belt.

Chapter iii contain ninety-four papers on the ore deposits of the Canadian Pre-Cambrian Shield. It is divided into ten parts as follows: "Yellowknife–Great Bear Lake Area," "Manitoba and Saskatchewan," "Red Lake–Patricia Area," "Little Long Lac–Sturgeon River Area," "Steep Rock–Michipicoten-Goudreau Area," "Porcupine-Ramore Area," "Sudbury Area," "Matachewan–Kirkland Lake–Larder Lake Area," "Rouyn-Chiboughamau Area (Western Part)," and "Rouyn-Chiboughamau Area (Eastern Part)."

In chapter iv there are five papers covering deposits in the eastern townships and Gaspe, and in Nova Scotia.

Structural Geology of Canadian Ore Deposits is clearly and concisely written. It is illustrated with numerous geologic maps and sections and with photographs, all of which are well executed and reproduced. For a volume which emphasizes structural features, however, there are many illustrations which lack pertinent structural data. Many of the maps show no strike, dip, or lineation symbols which might indicate why an ore body occupies its present position. That changes in structural attitude are commonly loci of ore bodies is known to every student of mineral deposits. Yet in many cases the structural data which would illustrate this are not given. This may be in part the result of company policy with regard to publication, but it is probably not true of all such omissions.

The volume is an excellent reference work, covering the important operating mines and mining districts of Canada. Although structural relationships are emphasized, the treatment in many of the articles is much broader and in-

cludes sections on general geology, mineralization, alteration, and genesis. The papers were written by 121 contributors especially for this symposium and have been published to commemorate the fiftieth anniversary of the founding of the Canadian Institute of Mining and Metallurgy. For the most part government geologists contributed the general papers, and mine or consulting geologists wrote the papers on individual mines. Some of the articles describe relatively new areas, and others present up-to-date information on older mines. The volume contains data previously widely scattered through the literature. In some cases the information is new. For example, to this reviewer's knowledge, here is the first detailed presentation of the modern views of the International Nickel Company's geologists on the origin of the Sudbury ore body. The information on the structure of the Sudbury irruptive and on the genesis of the ore is most welcome. It illustrates once again the advantgaes of having detailed information, especially when this is combined with the ability to interpret and generalize, before theorizing. The data on the Sullivan mine and on the West Bay fault are most welcome, to mention only two of the areas which previously had been treated inadequately in the literature.

In general, each district or region is treated as a unit and then the individual mines are described. It is to be regretted that some of the older nonoperating mines and mining districts have been omitted. An example is the Cobalt district, a discussion of which would have added to the understanding of the mineralization within the petrologic province which extends from the Duluth gabbro eastward into Quebec and includes the Michigan copper district, Sudbury, Cobalt, Silver Islet, and a number of other silver, copper, and pyrrhotite deposits.

It would have been helpful if the deposits had been classified according to the principal structural control, as was done by Newhouse in *Ore Deposits as Related to Structural Features*. This was done for certain of the deposits in the first section of the volume, but a more comprehensive discussion and tabulation would have emphasized the structural aspects with which the authors of the book were concerned.

As stated in the Preface, the book "is not presented as a completed undertaking but rather as a 'stock taking' of existing knowledge." The committee of the institute has achieved this purpose admirably. A vast amount of information has been assembled in one volume. The book should furnish many facts necessary for further work on Canadian ore deposits, such as: correlations of the structural relations of the various mining districts with the tectonic pattern of Canada, types of mineralization in the different metallogenetic epochs and provinces of Canada, types of ores related to wall-rock alteration, and the relation of structure to ore genesis. Much of the information required for such studies is not available, but a start has been made, and many references are given in this volume. Such studies should produce generalizations of considerable value to the science of mineral deposits.

The committee of the institute is to be congratulated for this comprehensive compilation of information. The officers of the Canadian mining companies are also commended for releasing hitherto unpublished data on individual mines.

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Geology: An Introduction to Earth History. By H. H. READ. ("Home University Library of Modern Knowledge," no. 198.) London, New York, and Toronto: Oxford University Press, 1949. Pp. 248; figs. 30. $2.00.

On the dust jacket of this book is the statement that, although all the authors in the "Home University Library" series are experts in their subjects, "they are not selected for their academic distinction alone; they are also skilled in the art of presenting their material and able to unite learning with lucidity." If this volume is typical of the series, it is no idle boast. Almost every geologist, at one time or another, has been asked by intelligent friends to suggest to them a book on geology. Whether their aim is to become familiar with the history of the earth, to acquire a knowledge of the processes that shape the earth, or to obtain a picture of the aims, methods, and scope of the science, this volume appears to fill the bill. It does not smack of the average elementary textbook, with its multitude of facts and choppy, often pedantic style, which one hesitates to recommend as leisure-time reading to an intelligent person. Although many of the fundamental problems of earth history are dealt with, the book demands no special pre-

liminary acquaintance with geology. Yet Dr. Read skilfully avoids the pitfall of making everything too pat. In urbane style he leads the reader gently, yet alertly, along, provoking and anticipating questions and adroitly introducing topics when they enter the reader's mind as the logical consequence of what went before.

The first of the five chapters, "The Search for the Method," gives the history of the succession of ideas which founded the science of geology and at the same time prepares the reader for the rest of the book It discusses the significance of fossils and of fossil-bearing beds, the growth of the ideas that rocks could be derived from pre-existing rocks and of correlation by fossils, and the realization that the principle of uniformitarianism is the cornerstone of geology. In other words, it presents the entry of the concepts of space and time into the study of rocks.

The second chapter, "The Sedimentary Rocks as Historical Documents,' treats two main problems: (1) the sedimentary rocks as revealing the environments in which they were laid down and (2) the succession of these rocks as revealing a sequence of environments. Under the first heading is a flow-sheet and adequate, though brief, discussion of the various sorts of sediments and their modes of deposition. Under the second are such topics as the great inundations, marine transgressions and regressions and their records, the question of the permanence of the ocean basins, a first statement of the divisions of the record of earth history, and a discussion of the methods used in estimating geologic time in years.

The chapter entitled "Interventions from the Depths" begins with a discussion of volcanic rocks and proceeds to an excellent presentation of the problems of regional metamorphism and granitization. Read divides the rocks into sedimentary, volcanic, and plutonic, the latter including granitic, migmatitic, and metamorphic rocks. The chapter continues with a discussion of the knowledge of the earth's interior from earthquake waves and of the origin of the earth, in which there is an unfortunate omission of recent theories for which the astronomical arguments are strong.

In chapter iv, "The Revolutionary Episodes," Read takes up what he considers "the short violent periods when the sediments were disturbed and land-masses extended and elevated," a concept that will not be palatable to many geologists. However, his discussion of the way in which these "earth revolutions" are recorded and how the record is read, and his presentation of joints, faults, folds, geosynclinal history, and isostasy is very good and is followed by an excellent discussion of the ultimate mechanism of diastrophism, in which the contraction, convection-current, and continental-drift hypotheses are examined pro and con, with no dogmatic final choice made.

Chapter v, "The Pattern of Earth History," weaves into a unified story the diverse strands which make up earth history. Here is a chapter that might have been merely a boring synopsis and repetition in "the seas came in, the seas went out" manner or a "Sunday supplement" recital of the wonders and oddities of the past. Instead, he presents a lucid and always interesting account of the true highlights of earth history, selecting only sufficient episodes to preserve the continuity. Of these he examines a few in some detail as illustrative of profound and significant matters in earth history, among them the special methods developed for reading the pre-Cambrian record, the pattern of lower Paleozoic life, and the Coal Measures rhythm. It might be expected that an English book would be unsuitable for the American reader because most of the examples would be taken from the geology of Great Britain or at least from Europe. However, the whole book and this final chapter in particular are not entirely the geology of Great Britain; they are world-wide in scope, with only a very slight, and wholly justifiable, emphasis on Read's own country. For instance, there is a long discussion of Finnish and Canadian pre-Cambrian records, of the history of the Appalachian geosyncline, of the Gondwana series of India and Africa, and of Jurassic and Cretaceous events in North America.

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Principles of Sedimentation. By W. H. TWENHOFEL. 2d ed. New York: McGraw-Hill Book Co., 1950. Pp. xii+673; figs. 81; tables 32. $6.50.

The second edition of this well-known text and reference work follows the first after a lapse of eleven years. It is larger by fifty-six pages, and there are nearly twice as many illustrations, including diagrams, photographs, and microphotographs. Among features newly illustrated are progressive and regressive marine overlap, soil erosion, microphotographs of sand grains, thin sections of coal, mud cracks, armored mud

ball, cone-in-cone, and stylolites. The pictures are well chosen and fairly well reproduced but still leave unillustrated many features which would make the book more understandable to the student. The bibliographic references are increased about 30 per cent with citations of more recent literature. The new illustrations, tables, and expanded bibliography occupy a little more than half the additional pages. The organization of the older book has been exactly followed, the principal parts of the book being (1) environments (2 chaps.); (2) origin of inorganic sediments (1 chap.); (3) interrelations of organisms and sediments (1 chap.); (4) transportation and deposition, including diagenesis (1 chap.); (5) classification (1 chap.); (6) description of types of sediments (6 chaps.); (7) structures, textures, and colors (2 chaps.). Only one new minor heading on selenium as a minor constituent of sediments was observed.

As the older edition of this book is familiar to most geologists, this review is largely concerned with the alterations in the new book. There are virtually no omissions of text from the first edition, but in numerous places sentences have been rewritten to achieve greater clarity of statement. The work is well proofread, and very few typographical errors were observed. A careful comparative reading of the two books resulted in finding over a hundred separate additional statements largely based on recent publications and on the author's personal researches, especially in the field of lake sediments. These are generally as brief as one to three sentences, and the largest single addition, a description of the lower Mississippi delta based on work by Russell and Russell, is two and a half pages long. Most of the new material is so well integrated into the text that it might be missed by the reader unless the two editions were being compared.

The author throughout displays the caution in the acceptance of new interpretations for which he is well known. He expresses open disapproval of some proposals, such as Russell's theory of loess formation, Shaub's theories for cone-in-cone and stylolites, Pettijohn's proposal to return to Grabau's endogenetic and exogenetic classes of rocks, and his proposal to amend the definition of graywacke so that it is not the basic equivalent of arkose. He withdraws support earlier given to Galliher's explanation of glauconite as an alteration product of biotite. He states (p. 549) that, although suspended loads of sediment may tend to settle in stratified arrangement, it is doubtful that this would occur in natural waters. Recently (1950) Kuenen has shown that sediment transported by turbidity currents may settle naturally in stratified fashon.

The interval since Twenhofel's first edition of *Principles of Sedimentation* has been a period of prolific output of significant new literature in sedimentation and related fields. There has been a more searching analysis of the operation of transportational agencies, a shift in emphasis from simple environments of sedimentation to the tectonic framework responsible for the pattern of environments. Studies in paleoecology have expanded rapidly, and increasing attention is being devoted to postdepositional changes in sediments. True petrographic methods are being increasingly applied to the products of sedimentation, and these are being treated more and more by quantitative methods of analysis. Geosynclines, facies, reefs, turbidity currents, periglacial phenomena, and phreatic versus vados origin of caverns have all become live subjects to which symposia, memoirs, and special numbers of geological journals have been devoted. It seems unwarranted under these conditions to adhere so strictly to the old organization and even more so to omit from consideration numerous of these newer subjects. This manuscript was completed for the printer early in 1948, and since that date there have appeared such works as (1) Pettijohn's *Sedimentary Rocks;* (2) Shrock's *Sequence of Layered Rocks;* (3) Shepard's *Submarine Geology;* (4) Trask's *Applied Sedimentation;* (5) The Geological Society of America's memoir on facies; (6) special issues of the *Journal of Geology* and the *Bulletin of the American Association of Petroleum Geologists* on reefs; (7) several papers by Krumbein, Sloss, and Dapples on tectonic controls on sedimentation; and (8) papers by Kay on types of geosynclines and cratonic platforms. These and many other recent contributions point to the need for future revision of this book to bring it up to date.

The reviewer searched in vain for references to, or excerpts from, numerous papers which he has considered outstanding contributions to sedimentation of the period from 1939 to 1947 which were presumably available to the author.

Thus in treating environments it is not indicated that glaciers may dissipate through stagnation as well as by retreat (Flint); there is no citation or reference to the excellent treatment of valley flats by Happ and Rittenhouse in "Some Principles of Accelerated Stream and Valley Sedimentation"; and, though Fisk's

name is used as a source of information on valley-flat sediments, there is no change in the text on this subject from the previous edition prior to the issuance of his monumental paper, *Geological Investigations of the Alluvial Valley of the Lower Mississippi River*. Under the cave environment no mention is made of Bretz's researches into limestone caverns. The periglacial environment and permafrost escape mention. Under "Lagoons" no mention is made of Lucke's work on Barnegat Bay and Inlet.

In treating transportation and deposition of sediments the outstanding book of Bagnold, *The Physics of Blown Sand and Desert Dunes*, is mentioned only in connection with maximum height of dunes, though this includes a splendid treatment of saltation in air currents, its initiation and maintenance. Saltation is mentioned only in connection with currents of water. In glacial transportation Demorest's contributions to glacial dynamics also escape mention.

In the classification and description of the various types of sediments, the traditional method of size grades for the clastics and chemical composition for the nonclastics is followed, and there is no mention of tectonic facies, contrasting sediments of actively subsiding basins with those of stable platforms or cratonic areas. This has been the most significant new trend in the treatment of sedimentary rocks and is the underlying principle in much of Pettijohn's *Sedimentary Rocks*. Undue space (p. 505) seems to be given to an interesting occurrence of doubly terminated quartz crystals in Permian gypsum for which the author offers no interpretation.

In summary the reviewer regards this book as an excellent useful reference work in sedimentation which concisely digests the content of several hundred research papers generally, with appraisal as to their relative importance, but he believes that the work suffers because the organization and content of the earlier edition have been adhered to closely, without adequate modification in the light of the multitude of important researches and new viewpoints of the past decade.

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Introduction to Theoretical Igneous Petrology. By ERNEST E. WAHLSTROM. New York: John Wiley & Sons, Inc., 1950. Pp. ix+365; figs. 150. $6.00.

This book starts with an introductory chapter and a long description (38 pp.) of the elementary concepts of heterogeneous equilibria. In my opinion this section is unnecessarily long, but it is in harmony with the spoon-feeding tradition of American teaching.

The section, "Equilibrium in Silicate Systems," is useful but is separated from that on "Igneous Minerals" for no good reason. Otherwise the chapters are well compiled. However, in the discussion of the feldspar system—the most important of all ternary rock-making systems—the consequences of the determination by Schairer and Bowen (1947) of the equilibrium relationships between potash feldspar and anorthite should have been emphasized. The discussion, as it stands, is misleading, and so, in consequence, is that of the potash-rich rocks (pp. 306 ff.).

Part 2 contains illuminating, but somewhat randomly arranged, chapters on the crust and the interior of the earth, on magmas, igneous differentiation, assimilation and syntexis, granitization, and origin of rock types, and closes with a brief survey of the classification of rocks. A short chapter on elementary physicochemical concepts is appended.

There is great need for a text covering the formation processes of all types of rock. Although Wahlstrom's textbook, as indicated by the title, covers only igneous rocks, it is the first of its kind in England. It is my hope that it will prepare the ground and induce American colleges and universities to teach less of the purely descriptive petrography and present, instead, what to me seems the more useful alternative in the education of both the academic and the professional geologist, namely, an examination and interpretation of the geological processes responsible for the formation of igneous rocks (and eventually of all rocks) in terms of physicochemical principles.

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COMMUNICATIONS AND ANNOUNCEMENTS

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Participants are urged to file titles and abstracts by June 15, 1951, in order to be assured of a place on the program. The final date is September 15. Abstracts are limited to 250 words and should be submitted in triplicate. No abstract will be printed unless the paper is presented orally, and no author will be allowed more than 15 minutes for presentation except by special request.

In addition to the general sessions in geology and geography, special programs on "The Crystalline Rocks of the Older Appalachians" and "Foreign Petroleum Areas" and possible joint meetings with other sections are being planned. The vice-presidential address will be given by Kenneth K. Landes.

Your co-operation in calling this meeting to the attention of your colleagues or other possible participants will be appreciated. Participation is not limited to members.

Titles and abstracts should be sent to: Leland Horberg, Secretary, Department of Geology, University of Chicago, Chicago 37, Illinois.

ERRATUM

In "Niagaran Reefs of the Great Lakes Area," by Heinz A. Lowenstam, vol. 58, no. 4, p. 443, l. 10 from bottom of first column, *read* The cavity fillings are therefore recent and diagenetic *instead of* The cavities and filling. . . .

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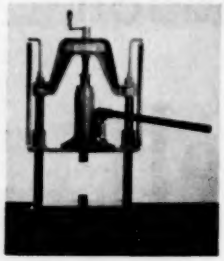

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